Analysis of spatial correlations between satellite-measured gravity anomalies and geological structures of the Earth’s mantle

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Motivations

Why such an analysis?

- Basic idea: the use of satellite gravity observations to refine Earth’s geophysical models

![Diagram of gravity analysis](image-url)

- Observed gravity values
- Modelled gravity values
- Differences in gravity values
- Forward modelling
- Inverse modelling
- Earth’s geophysical model: shape, location, density of geological structures
Motivations

Why such an analysis?

The best model is obtained when $\|\varepsilon\|$ is minimal

- Observed gravity values
- Modelled gravity values
- Differences $\varepsilon$
- Observed-modelled gravity values
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- Inverse modelling
- Earth's geophysical model
  shape, location, density of geological structures
Motivations

Why such an analysis?

- For inversion to be possible from an initial model, $\|\varepsilon\|$ must not be too large.

![Diagram showing forward and inverse modelling process]

- **Observed gravity values**
- **Modelled gravity values**
- **Differences** $\varepsilon$
- **Observed-modelled gravity values**

**Forward modelling**

**Inverse modelling**

**Earth's geophysical model**

*shape, location, density* of geological structures
Motivations

Our own experiments

• Comparison of GOCE-derived gravity gradients at GOCE altitude with their modelling values from two Earth’s geophysical models

- Observed gravity values
- Global gravity gradients derived from GOCE measurements
- Modelled gravity values
- Differences \( \sum \) Observed-modelled gravity values

Forward modelling
Inverse modelling

Earth's geophysical model
shape, location, density of geological structures

1. LITHO1.0/PREM combined model
2. S40RTS model
LITHO1.0
Available at 1°x1° resolution

1st Earth’s geophysical model: LITHO1.0 ~ PREM

LITHO1.0
Available at 1°x1° resolution

PREM
Dziewonski and Anderson, 1981

Combined model

**LITHO1.0: physical parameters**

<table>
<thead>
<tr>
<th>num</th>
<th>layer</th>
<th>parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ice</td>
<td>(thickness), (vp), (vs), (ρ), (Q)</td>
</tr>
<tr>
<td>2</td>
<td>water</td>
<td>(thickness), (vp), (vs), (ρ), (Q)</td>
</tr>
<tr>
<td>3</td>
<td>sediment layer 1</td>
<td>(thickness), (vp), (vs), (ρ), (Q)</td>
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<tr>
<td>4</td>
<td>sediment layer 2</td>
<td>(thickness), (vp), (vs), (ρ), (Q)</td>
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<tr>
<td>5</td>
<td>sediment layer 3</td>
<td>(thickness), (vp), (vs), (ρ), (Q)</td>
</tr>
<tr>
<td>6</td>
<td>upper crust</td>
<td>thickness, vp, vs, ρ, (Q)</td>
</tr>
<tr>
<td>7</td>
<td>middle crust</td>
<td>thickness, vp, vs, ρ, (Q)</td>
</tr>
<tr>
<td>8</td>
<td>lower crust</td>
<td>thickness, vp, vs, ρ, (Q)</td>
</tr>
<tr>
<td>9</td>
<td>lithospheric mantle (lid)</td>
<td>thickness, vp, vs, (ρ), (Q)</td>
</tr>
<tr>
<td>10</td>
<td>asthenospheric mantle</td>
<td>vp, vs, (ρ), (Q)</td>
</tr>
</tbody>
</table>

*a* Parameters unmodified from the starting model are shown in parentheses.

**Starting models**

- **CRUST1.0**  
  Laske et al., 2012
- **LLNL-G3D**  
  Simmons et al., 2012  
  Pasyanos, 2010 (thickness)  
  Kennett et al., 1995
1st Earth’s geophysical model: LITHO1.0

Maximum depth of geological structures

km

IUGG 2019 Meeting, Montreal (Qc) Theory and Methods of Potential Fields JG02b July 16th 2019
Software used: **Tesseroids**\(^1\)

Computation of gravity gradients

Software used: Tesseroids


- Numerical integration based on the Gauss-Legendre quadrature
Computation of gravity gradients

Software used: **Tesseroids**


- Numerical integration based on the Gauss-Legendre quadrature
- Earth’s solid body decomposed into *spherical prisms*
Computation of gravity gradients

- Direct direct computation of the gravity gradients induced by each constant density ($\rho$) prism in the LNOF associated with GOCE
- $\lambda_2 - \lambda_1 = 1^\circ$; $\varphi_2 - \varphi_1 = 1^\circ$; $\rho$, $r_2 - r_1 \leftarrow \text{LITHO1.0}$
- Computation by means of analytical formulas for PREM (ellipsoidal concentric layers)

![Spherical prism]

Original figure from: GOCE Level 2 Product Data Handbook, EGG-C, 2014
Computation of gravity gradients
Computational considerations
LITHO1.0 $\rightarrow$ **447 824** tesserialds (spherical prims)
Computation of gravity gradients
Computational considerations

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- Worldwide $1^\circ \times 1^\circ$ grid at GOCE altitude (255 km a.s.l) → \textbf{64 800} computation points
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- **6** gravity gradients per point
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Computation of gravity gradients
Computational considerations

174 113 971 200 computations based on Gauss quadrature performed in 8 hours using a computing cluster consisting of 5 computing servers (nodes) totalling:

- 200 cores
- 336 GB of memory
- 36.1 TB of storage capacity in a gigabit network
Anomaly of gravity gradient $V_{ij}, i, j = x, y, z$

$$\Delta V_{ij, EM} = V_{ij, EM} - V_{ij, PREM}$$

$$\Delta V_{ij, GOCE} = V_{ij, GOCE} - V_{ij, PREM}$$

where EM = "Earth’s geophysical Model", GOCE = "GOCE data" and PREM = "PREM model"
Anomaly of GOCE $V_{zz}$ measurement/PREM (255 km a.s.l)

Anomaly of $V_{zz}$ computed from the combined model LITHO1.0 $\sim$ PREM/PREM (255 km a.s.l)
Comments on the results

- The variation range of the calculated gravity gradient values is about 5 times greater than that of the GOCE-derived values.
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- $\Rightarrow$ The differences ($\varepsilon$) between observed gravity gradient values and modelled ones are too high for performing an inversion.
- $\Rightarrow$ Substantial adjustments of the model first have to be done before going further in model refinement.
Comments on the results

- The variation range of the calculated gravity gradient values is about \( 5 \) times greater than that of the GOCE-derived values.
- \( \Rightarrow \) The differences (\( \varepsilon \)) between observed gravity gradient values and modelled ones are too high for performing an inversion.
- \( \Rightarrow \) Substantial adjustments of the model first have to be done before going further in model refinement.
- Critical issue: regardless of model accuracy in terms of density values, the coordinates of the points limiting the geological structures must be expressed in the same reference frame as the one used to define GOCE-derived gravity gradients.
Model of shear-velocity variations in Earth’s mantle deduced from seismic data

Model of shear-velocity variations in Earth’s mantle deduced from seismic data
Extended from 100 km to 2836 km depth

Earth’s geophysical model: S40RTS

- Model of shear-velocity variations in Earth’s mantle deduced from seismic data
- Extended from 100 km to 2836 km depth
- The relationship between the relative variation in shear-velocity ($V_s$) and density ($\rho$) is given by:

$$\frac{\Delta \rho}{\rho} = a \frac{\Delta V_s}{V_s}$$

where $a = 0.2$ or $0.3$ according to Karato’s estimates (1993, 2001)


2nd Earth’s geophysical model: S40RTS
Density distribution as a function of depth

Density distribution at 100 km depth

\[ a = 0.2 \]

\[ a = 0.3 \]
2nd Earth’s geophysical model: S40RTS
Density distribution as a function of depth

Density distribution at 600 km depth

\[ \alpha = 0.2 \]

\[ \alpha = 0.3 \]

Density distribution at 600 km depth
2nd Earth’s geophysical model: S40RTS

Density distribution as a function of depth

Density distribution at **1536 km** depth

\[ a = 0.2 \]  
\[ a = 0.3 \]  

\[ \text{kg/m}^3 \]
Density distribution at 2836 km depth

\[ a = 0.2 \]

\[ a = 0.3 \]

⇒ The density distributions are significantly different depending on the value of the parameter \( a \)
The 2nd Earth’s geophysical model: S40RTS

$V_{zz}$ gravity gradient resulting from S40RTS-deduced density distribution

Computation involving $1,739,883$ tessoids, thus giving $676,466,510,400$ computations for calculating 6 gravity gradients worldwide $\Rightarrow$

computation time: 24 hours
Major differences in the range of $V_{zz}$ values:

$|V_{zz}^{\text{Max}} - V_{zz}^{\text{Min}}| = 10 \ E \ (a = 0.2)$ and $|V_{zz}^{\text{Max}} - V_{zz}^{\text{Min}}| = 46 \ E \ (a = 0.3)$
Conclusions and prospects
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1. Direct comparison of the worldwide gravity gradients calculated from Earth’s geophysical models and those derived from GOCE measurements is to date still tricky.

2. Geo-referencing of Earth’s geophysical models is a fundamental issue in order to assimilate satellite-derived gravity gradients into the models.
Conclusions and prospects

1. Direct comparison of the worldwide gravity gradients calculated from Earth's geophysical models and those derived from GOCE measurements is to date still tricky.

2. Geo-referencing of Earth’s geophysical models is a fundamental issue in order to assimilate satellite-derived gravity gradients into the models.

3. Close collaboration between geodesians and geophysicists must be initiated with a view to exploit all valuable gravity data acquired by satellites.
Thank you for your kind attention!