



HAL
open science

Robust Sensor Fault Detection for Linear Parameter-Varying Systems using Interval Observer

Thomas Chevet, Thach Ngoc Dinh, Julien Marzat, Tarek Raïssi

► **To cite this version:**

Thomas Chevet, Thach Ngoc Dinh, Julien Marzat, Tarek Raïssi. Robust Sensor Fault Detection for Linear Parameter-Varying Systems using Interval Observer. 31st European Safety and Reliability Conference, Sep 2021, Angers, France. pp.1486-1493, 10.3850/978-981-18-2016-8_380-cd . hal-03239385

HAL Id: hal-03239385

<https://cnam.hal.science/hal-03239385>

Submitted on 15 Feb 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Robust Sensor Fault Detection for Linear Parameter-Varying Systems using Interval Observer

Thomas Chevet

Cedric, Conservatoire National des Arts et Métiers, Paris, France.

DTIS, ONERA, Université Paris-Saclay, 91123 Palaiseau, France. E-mail: thomas.chevet@onera.fr

Thach Ngoc Dinh

Cedric, Conservatoire National des Arts et Métiers, Paris, France. E-mail: ngoc-thach.dinh@lecnam.net

Julien Marzat

DTIS, ONERA, Université Paris-Saclay, 91123 Palaiseau, France. E-mail: julien.marzat@onera.fr

Tarek Raïssi

Cedric, Conservatoire National des Arts et Métiers, Paris, France. E-mail: tarek.raïssi@cnam.fr

This paper proposes a new interval observer for continuous-time linear parameter-varying systems with an unmeasurable parameter vector subject to unknown but bounded disturbances. The parameter-varying matrices are assumed to be elementwise bounded. This observer is used to compute a so-called residual interval used for sensor fault detection by checking if zero is contained in the interval. To attenuate the effect of the system's uncertainties on the detectability of faults, additional weighting matrices and different upper and lower observer gains are introduced, providing more degrees of freedom than the classical interval observer strategies. In addition, a L_∞ procedure is proposed to tune the value of the observer gains, this procedure being easy to modify to introduce additional constraints on the estimation algorithm. Simulations are run to show the efficiency of the proposed fault detection strategy.

Keywords: Fault detection, Robust fault detection, Sensor fault, Linear Parameter-Varying system, Interval observer, Continuous-time systems.

1. Introduction

Most real-life systems obey nonlinear dynamics, making the design of control and estimation algorithms a complex task. A common and powerful way to reduce this complexity is to use a linear parameter-varying (LPV) plant (Shamma, 2012). Indeed, due to their partial linearity, methods developed for linear systems can be applied to such plants. In addition, during their evolution, systems can be subject to faults that could provoke serious damages if they go undetected. Therefore, strategies have to be developed to detect such faults. The case of sensor fault detection has been widely studied in the literature (Varga, 2017). Passive fault detection strategies usually rely on the comparison of the system's output with the estimate of the output computed from a fault-free model (Li et al., 2020). However, when the system is subject to unknown but bounded disturbances, classical strategies are limited (Robinson et al., 2020), difficult to implement and could lead to a false positive (Lamouchi et al., 2018).

Set-based estimation algorithms have been intro-

duced to deal with the problem of state estimation for uncertain systems, provided that the uncertainties are bounded. Two classes of algorithms have been developed: set-membership estimation (Combastel, 2005; Li et al., 2020) and interval observers (Mazenc and Bernard, 2011; Raïssi et al., 2011; Wang et al., 2018; Garbouj et al., 2020). In this paper, interval observers are considered to design a passive sensor fault detection strategy. While a classical pointwise observer computes an estimate of the true value of the system's state based on the system's dynamics and output, an interval observer uses two sub-observers to provide bounds for the system's state by also taking into account the uncertainties' bounds. The advantage of such estimation strategies is that they are often more computationally efficient than set-membership algorithms.

Several passive set-based sensor fault detection strategies for LPV systems have been proposed, both based on set-membership estimation (Nejjari et al., 2009; Blesa et al., 2012; Wan et al., 2020) and interval observers (Lamouchi et al., 2018;

Rotondo et al., 2018, 2019; Ifqir et al., 2020; Garbouj et al., 2020). The classical approach consists in computing a so-called residual interval. If the interval does not contain zero, then the system is subject to a fault. However, due to this strategy, low-magnitude faults could go undetected. Therefore, to ensure that the maximum range of faults is detected, the residual interval has to be tight. Interval observers are based on a change of coordinates meant to ensure that they satisfy the cooperativity property (Mazenc and Bernard, 2011) (i.e. the estimation error state matrix is Metzler and the estimation error dynamics are stable). The performance of the fault detection algorithm (i.e. the tightness of the residual interval) is then heavily influenced by the choice of target coordinates. For this reason, Wang et al. (2018) propose the so-called TNL design strategy (where T , N and L denote the weighting matrices and gain used in this strategy), based on the introduction of additional weighting matrices in the observer design. This approach provides more degrees of freedom in the observer design since a change of coordinates is no longer necessary to ensure cooperativity. In addition, several interval observers for LPV systems have been proposed under the assumption that the vector of scheduling parameters is always available (Wang et al., 2015; Li et al., 2019). In the general case, only the bounds of the parameters (and thus of the parameter-varying matrices) are known (Efimov et al., 2012, 2013). This is why Chebotarev et al. (2015) introduce an interval observer with different gains for the upper and lower sub-observers.

Following Wang et al. (2018), Chebotarev et al. (2015) and Zammali et al. (2021), this study therefore proposes a sensor fault detection strategy for LPV systems based on a robust interval observer. The contributions of this paper are twofold: (i) a novel interval observer structure for continuous-time LPV systems, based on the TNL approach and used to build a residual interval, is introduced; (ii) a new modular L_∞ gain design procedure, ensuring the cooperativity of the estimation error dynamics and the tightness of the residual interval, is proposed.

The remainder of this paper is organized as follows. Section 2 presents general prerequisites and formulates the problem addressed. Section 3 introduces the proposed structure and design procedure for the interval observer. Simulation results are given in Section 4 to assess the efficiency of the proposed fault detection strategy. Finally, Section 5 draws concluding remarks and perspectives.

2. Prerequisites and problem formulation

2.1. Notations

The sets of positive integers, real numbers and positive real numbers are denoted respectively by \mathbb{N} , \mathbb{R} and \mathbb{R}_+ . The matrices I_n and $\mathbf{0}$ are

respectively the identity matrix of size $n \in \mathbb{N}$ and the matrix filled with zeros of appropriate size. The matrices A^\top and A^\dagger , with $A \in \mathbb{R}^{n \times m}$, denote respectively the transpose and the Moore-Penrose inverse of matrix A . The fact that a matrix $A \in \mathbb{R}^{n \times n}$ is positive definite (resp. semidefinite) is denoted by $A \succ 0$ (resp. $A \succeq 0$) and the fact that A is negative semidefinite is denoted by $A \preceq 0$. The matrix $\text{diag}(A_1, \dots, A_n)$ is the block diagonal matrix with diagonal blocks A_1, \dots, A_n . Given a signal $x : \mathbb{R}_+ \rightarrow \mathbb{R}^n$, x_t denotes the value of x at time t (the notation $x(t)$ is alternatively used in the literature). The Euclidean norm of x_t is $\|x_t\|_2^2 = x_t^\top x_t$ and the L_∞ norm of x is $\|x\|_\infty = \sup\{\|x_t\|_2, t \in \mathbb{R}_+\}$. The set of all signals x satisfying $\|x\|_\infty < \infty$ is denoted by \mathcal{L}_∞^n . The Kronecker product of two matrices A and B is denoted by $A \otimes B$. Finally, \star is a placeholder denoting the transpose of a term placed symmetrically in a matrix.

2.2. Interval analysis

Let $A, B \in \mathbb{R}^{n \times m}$ be two matrices. Then the inequality $A \leq B$ is understood elementwise. The matrix A can be written as $A = A^+ - A^-$, with $A^+ = \max\{\mathbf{0}, A\}$ (the maximum operator is understood elementwise), such that $A^+, A^- \geq 0$. The above operations are extended without modification to any vector $x \in \mathbb{R}^n$.

Lemma 2.1 (Efimov et al. (2012)). *Let $x, \underline{x}, \bar{x} \in \mathbb{R}^n$ be vectors satisfying $\underline{x} \leq x \leq \bar{x}$.*

(i) *Let $A \in \mathbb{R}^{m \times n}$ be a constant matrix. Then:*

$$A^+ \underline{x} - A^- \bar{x} \leq Ax \leq A^+ \bar{x} - A^- \underline{x}.$$

(ii) *Let $A, \underline{A}, \bar{A} \in \mathbb{R}^{m \times n}$ be matrices such that $\underline{A} \leq A \leq \bar{A}$. Then:*

$$\begin{aligned} \underline{A}^+ \underline{x}^+ - \bar{A}^+ \underline{x}^- - \underline{A}^- \bar{x}^+ + \bar{A}^- \bar{x}^- &\leq Ax \\ &\leq \bar{A}^+ \bar{x}^+ - \underline{A}^+ \bar{x}^- - \bar{A}^- \underline{x}^+ + \underline{A}^- \underline{x}^-. \end{aligned}$$

Remark 2.1. If $x \in \mathbb{R}^n$ is a constant vector and $\underline{A}, \bar{A} \in \mathbb{R}^{m \times n}$ are matrices such that $\underline{A} \leq A \leq \bar{A}$, then, by (i) of Lemma 2.1:

$$\underline{A}x^+ - \bar{A}x^- \leq Ax \leq \bar{A}x^+ - \underline{A}x^-.$$

In addition to the above results, it is necessary to introduce a particular kind of matrix for the development of interval observers for continuous-time systems.

Definition 2.1 (Chebotarev et al. (2015)). *A matrix $A \in \mathbb{R}^{n \times n}$ is called Metzler if all its off-diagonal elements are nonnegative. The matrix A is Metzler if there exists a diagonal matrix $D \in \mathbb{R}_+^{n \times n}$ such that $A + D \geq \mathbf{0}$.*

Assumption 3.1. *The pair (TA_0, C) is detectable.*

Inspired by Li et al. (2019), the proposed framer for the state of system (1) is:

$$\begin{cases} \dot{\underline{\xi}}_t = (TA_0 - \underline{L}C) \underline{x}_t + TB_0 u_t \\ \quad + \underline{L}y_t + \underline{\phi}_t + \underline{\chi}_t + \underline{\omega}_t \end{cases} \quad (6a)$$

$$\underline{x}_t = \underline{\xi}_t + Ny_t \quad (6b)$$

$$\begin{cases} \dot{\bar{\xi}}_t = (TA_0 - \bar{L}C) \bar{x}_t + TB_0 u_t \\ \quad + \bar{L}y_t + \bar{\phi}_t + \bar{\chi}_t + \bar{\omega}_t \end{cases} \quad (6c)$$

$$\bar{x}_t = \bar{\xi}_t + Ny_t \quad (6d)$$

where:

$$\begin{cases} \underline{\phi}_t = T^+ \delta_t(A, x) - T^- \bar{\delta}_t(A, x) \\ \bar{\phi}_t = T^+ \bar{\delta}_t(A, x) - T^- \delta_t(A, x) \end{cases} \quad (7)$$

$$\begin{cases} \underline{\chi}_t = T^+ (\underline{\Delta B}u_t^+ - \bar{\Delta B}u_t^-) \\ \quad - T^- (\bar{\Delta B}u_t^+ - \underline{\Delta B}u_t^-) \\ \bar{\chi}_t = T^+ (\bar{\Delta B}u_t^+ - \underline{\Delta B}u_t^-) \\ \quad - T^- (\underline{\Delta B}u_t^+ - \bar{\Delta B}u_t^-) \end{cases} \quad (8)$$

$$\begin{cases} \underline{\omega}_t = (TD_0)^+ \underline{\omega}_t - (TD_0)^- \bar{\omega}_t \\ \quad + T^+ \delta_t(D, w) - T^- \bar{\delta}_t(D, w) \\ \bar{\omega}_t = (TD_0)^+ \bar{\omega}_t - (TD_0)^- \underline{\omega}_t \\ \quad + T^+ \bar{\delta}_t(D, w) - T^- \delta_t(D, w) \end{cases} \quad (9)$$

with:

$$\begin{cases} \delta_t(M, a) = \underline{\Delta M}^+ a_t^+ - \bar{\Delta M}^+ a_t^- \\ \quad - \underline{\Delta M}^- a_t^+ + \bar{\Delta M}^- a_t^- \\ \bar{\delta}_t(M, a) = \bar{\Delta M}^+ a_t^+ - \underline{\Delta M}^+ a_t^- \\ \quad - \bar{\Delta M}^- a_t^+ + \underline{\Delta M}^- a_t^- \end{cases} \quad (10)$$

and \underline{L} and \bar{L} are observer gains such that $TA_0 - \underline{L}C$ and $TA_0 - \bar{L}C$ are Metzler. The matrices T and N are weighting matrices introduced in Wang et al. (2018) to add more degrees of freedom when tuning the values of \underline{L} and \bar{L} .

Theorem 3.1. *Let Assumptions 2.2 and 2.3 hold and $TA_0 - \underline{L}C$ and $TA_0 - \bar{L}C$ be Metzler matrices. Then, \underline{x}_t and \bar{x}_t obeying the dynamics (6) satisfy, in the fault-free case (i.e. for $f_t = \mathbf{0}$):*

$$\underline{x}_t \leq x_t \leq \bar{x}_t, \forall t \in \mathbb{R}_+. \quad (11)$$

Proof. In the following, it is assumed that $f_t = \mathbf{0}$, $\forall t \in \mathbb{R}_+$. Let $\underline{e}_t^\xi = \underline{\xi}_t - Tx_t$, $\bar{e}_t^\xi = \bar{\xi}_t - Tx_t$,

$\underline{e}_t = \underline{x}_t - x_t$ and $\bar{e}_t = \bar{x}_t - x_t$. Then:

$$\begin{aligned} \dot{\underline{e}}_t^\xi &= (TA_0 - \underline{L}C) \underline{x}_t + TB_0 u_t + \underline{L}y_t \\ &\quad + \underline{\phi}_t + \underline{\chi}_t + \underline{\omega}_t - TA(\rho_t)x_t \\ &\quad - TB(\rho_t)u_t - TD(\rho_t)w_t \\ &= (TA_0 - \underline{L}C) \underline{e}_t + \underline{\Phi}_t + \underline{X}_t + \underline{\Omega}_t \end{aligned}$$

where $\underline{\Phi}_t = \underline{\phi}_t - T\Delta A(\rho_t)x_t$, $\underline{X}_t = \underline{\chi}_t - T\Delta B(\rho_t)u_t$ and $\underline{\Omega}_t = \underline{\omega}_t - TD(\rho_t)w_t$. In addition:

$$\begin{aligned} \underline{e}_t &= \underline{\xi}_t + Ny_t - (T + NC)x_t \\ &= \underline{\xi}_t - Tx_t + Nf_t = \underline{e}_t^\xi. \end{aligned}$$

Therefore:

$$\dot{\underline{e}}_t = (TA_0 - \underline{L}C) \underline{e}_t + \underline{\Phi}_t + \underline{X}_t + \underline{\Omega}_t. \quad (12)$$

Following the same procedure:

$$\dot{\bar{e}}_t = (TA_0 - \bar{L}C) \bar{e}_t + \bar{\Phi}_t + \bar{X}_t + \bar{\Omega}_t \quad (13)$$

where $\bar{\Phi}_t = \bar{\phi}_t - T\Delta A(\rho_t)x_t$, $\bar{X}_t = \bar{\chi}_t - T\Delta B(\rho_t)u_t$ and $\bar{\Omega}_t = \bar{\omega}_t - TD(\rho_t)w_t$.

By Assumptions 2.2 and 2.3 and Lemma 2.1, $\underline{X}_t \leq \mathbf{0}$, $\bar{X}_t \geq \mathbf{0}$, $\underline{\Omega}_t \leq \mathbf{0}$ and $\bar{\Omega}_t \geq \mathbf{0}$. To prove that $\underline{x}_t \leq x_t \leq \bar{x}_t$, it is enough to prove that whenever the j -th component of \underline{e}_t (respectively \bar{e}_t) is equal to 0, the j -th component of the derivative $\dot{\underline{e}}_t$ (resp. $\dot{\bar{e}}_t$) is nonpositive (resp. nonnegative). Indeed, in this case, \underline{e}_t (resp. \bar{e}_t) can never become positive (resp. negative).

Let a_t^j denote the j -th component of a_t , where a can be any of the vector previously defined. If $\underline{e}_t^j = 0$ and $\bar{e}_t^j \geq 0$, then $\underline{x}_t^j = x_t^j \leq \bar{x}_t^j$ and, by Lemma 2.1, $\underline{\Phi}_t^j \leq 0$. Moreover:

$$\dot{\underline{e}}_t^j = \sum_{i=1}^{n_x} (TA_0 - \underline{L}C)_{ji} \underline{e}_t^i + \underline{\Phi}_t^j + \underline{X}_t^j + \underline{\Omega}_t^j$$

where $(TA_0 - \underline{L}C)_{ji}$ is the component located at the j -th row and i -th column of $TA_0 - \underline{L}C$. Considering $TA_0 - \underline{L}C$ is Metzler, by Definition 2.1, $(TA_0 - \underline{L}C)_{ji} \underline{e}_t^i \leq 0$, $\forall i \in \bar{1}, n_x$ such that $i \neq j$.

Since $\underline{e}_t^j = 0$, it does not contribute to $\dot{\underline{e}}_t^j$ and $\dot{\underline{e}}_t^j \leq 0$. The same reasoning on \bar{e}_t^j leads to $\dot{\bar{e}}_t^j \geq 0$.

With what precedes and the fact that, by Assumption 2.2, $\underline{\Phi}_0 \leq \mathbf{0}$, $\bar{\Phi}_0 \geq \mathbf{0}$, $\underline{e}_0 \leq \mathbf{0}$ and $\bar{e}_0 \geq \mathbf{0}$, (11) is satisfied. \square

The desired residual framer (4) can then be constructed from the proposed state framer (6).

3.2. Stability and L_∞ performance

For the framer (6) to be an interval observer, it is necessary that the estimation errors \underline{e}_t and \bar{e}_t are input-to-state stable (ISS). Accordingly, to guarantee input-to-state stability and to attenuate the effect of the uncertainties on the residual framer, as per the design requirements stated in Section 2.3, a L_∞ procedure is proposed to tune the observer gains \underline{L} and \bar{L} . To unroll this procedure, the following lemma is recalled.

Lemma 3.1 (Rao and Mitra (1972)). *Given matrices $X \in \mathbb{R}^{n \times m}$, $Y \in \mathbb{R}^{m \times p}$ and $Z \in \mathbb{R}^{n \times p}$ with $\text{rank} Y = p$, the general solution X of the equation $XY = Z$ is:*

$$X = ZY^\dagger + \Xi (I_m - YY^\dagger)$$

where $\Xi \in \mathbb{R}^{n \times m}$ is an arbitrary matrix.

From this lemma and the relations (5):

$$T = \Theta^\dagger \lambda_1 + \Xi \Psi \lambda_1, \quad N = \Theta^\dagger \lambda_2 + \Xi \Psi \lambda_2, \quad (14)$$

with $\Xi \in \mathbb{R}^{n_x \times (n_x + n_y)}$ a free matrix and:

$$\Theta = \begin{bmatrix} I_{n_x} \\ C^\top \end{bmatrix}, \quad \Psi = I_{n_x + n_y} - \Theta \Theta^\dagger,$$

$$\lambda_1 = \begin{bmatrix} I_{n_x} \\ \mathbf{0} \end{bmatrix}, \quad \lambda_2 = \begin{bmatrix} \mathbf{0} \\ I_{n_y} \end{bmatrix}.$$

The following theorem then presents the gain tuning procedure.

Theorem 3.2. *Let all the assumptions of Theorem 3.1 hold. For two given positive scalars α and η , if there exist two positive scalars γ and μ , a positive definite diagonal matrix $P \in \mathbb{R}^{2n_x \times 2n_x}$ and a block diagonal matrix $Y \in \mathbb{R}^{2n_x \times 2n_y}$ such that:*

$$S + \eta P \geq \mathbf{0} \quad (15a)$$

$$\begin{bmatrix} \Lambda_{11} & \star & \star \\ P & -\gamma I_{2n_x} & \star \\ P & \mathbf{0} & -\gamma I_{2n_x} \end{bmatrix} \preceq \mathbf{0} \quad (15b)$$

$$\begin{bmatrix} P & \star & \star \\ \mathbf{0} & \mu - \gamma & \star \\ C & \mathbf{0} & \mu I_{2n_y} \end{bmatrix} \succeq \mathbf{0} \quad (15c)$$

where $\Lambda_{11} = S + S^\top + \alpha P + \gamma Q$, $S = P(I_2 \otimes T)(I_2 \otimes A_0) - Y\Upsilon$, $\Upsilon = I_2 \otimes C$, $Q = 6 \cdot \text{diag}(\bar{l}_\phi^2, \bar{l}_\phi^2)$ and:

$$C = \begin{bmatrix} C^+ & -C^- \\ -C^- & C^+ \end{bmatrix},$$

then, (6) is a robust interval observer for system (1) in the fault-free case. This interval observer has the performance:

$$\|R_t\|_2^2 \leq \mu V_0 e^{-\alpha t} + \mu^2 \|\varepsilon\|_\infty^2 \quad (16)$$

where $R_t^\top = [r_t^\top \bar{r}_t^\top]$, $V_0 = E_0^\top P E_0$, $E_t^\top = [e_t^\top \bar{e}_t^\top]$ and:

$$\varepsilon_t = \begin{bmatrix} \underline{X}_t + \underline{\Omega}_t \\ \bar{X}_t + \bar{\Omega}_t \end{bmatrix}.$$

Proof. In the following, it is assumed that $f_t = \mathbf{0}$. Since $P \succ 0$, all its diagonal terms are strictly positive. Considering $Y = P \text{diag}(\underline{L}, \bar{L})$, the matrices $TA_0 - \underline{L}C$ and $TA_0 - \bar{L}C$ are Metzler if condition (15a) is satisfied, according to Definition 2.1.

The dynamics of E_t are:

$$\dot{E}_t = \Pi E_t + \Phi_t + \varepsilon_t \quad (17)$$

where $\Pi = (I_2 \otimes T)(I_2 \otimes A_0) - LY$, $L = \text{diag}(\underline{L}, \bar{L})$ and $\Phi_t^\top = [\Phi_t^\top \bar{\Phi}_t^\top]$.

The functions $T\Delta A(\rho_t)x_t$, ϕ_t and $\bar{\phi}_t$ satisfy the assumptions of Lemma 2.2 so that $\underline{\Phi}_t$ and $\bar{\Phi}_t$ are globally Lipschitz. In addition, by Lemma 2.2:

$$\begin{cases} \underline{\Phi}_t^\top \underline{\Phi}_t \leq (\bar{l}_\phi \|\bar{x}_t - x_t\|_2 + \underline{l}_\phi \|\underline{x}_t - x_t\|_2 + \underline{m}_\phi)^2 \\ \bar{\Phi}_t^\top \bar{\Phi}_t \leq (\bar{l}_\phi \|\bar{x}_t - x_t\|_2 + \underline{l}_\phi \|\underline{x}_t - x_t\|_2 + \bar{m}_\phi)^2 \end{cases}$$

where $\|\bar{x}_t - x_t\|_2^2 = \bar{e}_t^\top \bar{e}_t$ and $\|\underline{x}_t - x_t\|_2^2 = \underline{e}_t^\top \underline{e}_t$. Then, by Cauchy-Schwarz inequality:

$$\begin{cases} \underline{\Phi}_t^\top \underline{\Phi}_t \leq 3(\bar{l}_\phi^2 \bar{e}_t^\top \bar{e}_t + \underline{l}_\phi^2 \underline{e}_t^\top \underline{e}_t + \underline{m}_\phi^2) \\ \bar{\Phi}_t^\top \bar{\Phi}_t \leq 3(\bar{l}_\phi^2 \bar{e}_t^\top \bar{e}_t + \underline{l}_\phi^2 \underline{e}_t^\top \underline{e}_t + \bar{m}_\phi^2) \end{cases}$$

so that:

$$\Phi_t^\top \Phi_t \leq E_t^\top Q E_t + \beta \quad (18)$$

with $\beta = 3(\underline{m}_\phi^2 + \bar{m}_\phi^2)$.

Consider now the candidate Lyapunov function $V_t = E_t^\top P E_t$. The time derivative of V is:

$$\begin{aligned} \dot{V}_t &= \dot{E}_t^\top P E_t + E_t^\top P \dot{E}_t \\ &= E_t^\top (S^\top + S) E_t + \Phi_t^\top P E_t + E_t^\top P \Phi_t \\ &\quad + E_t^\top P \varepsilon_t + \varepsilon_t^\top P E_t \\ &= E_t^\top (S^\top + S + \alpha P) E_t + \Phi_t^\top P E_t \\ &\quad + E_t^\top P \Phi_t + E_t^\top P \varepsilon_t + \varepsilon_t^\top P E_t \\ &\quad - \alpha E_t^\top P E_t + \gamma \Phi_t^\top \Phi_t - \gamma \Phi_t^\top \Phi_t \\ &\quad + \gamma \varepsilon_t^\top \varepsilon_t - \gamma \varepsilon_t^\top \varepsilon_t \end{aligned}$$

so that, with (18):

$$\dot{V}_t \leq \begin{bmatrix} E_t \\ \Phi_t \\ \varepsilon_t \end{bmatrix}^\top \Lambda \begin{bmatrix} E_t \\ \Phi_t \\ \varepsilon_t \end{bmatrix} - \alpha V_t + \gamma \|\varepsilon_t\|_2^2 + \gamma \beta \quad (19)$$

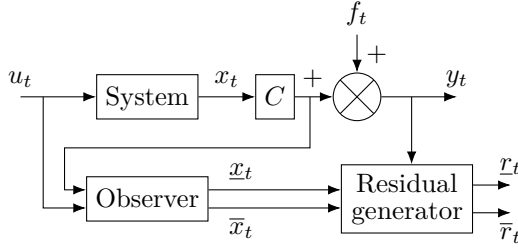


Figure 1. Structure of the fault detection strategy.

where:

$$\Lambda = \begin{bmatrix} \Lambda_{11} & P & P \\ P & -\gamma I_{2n_x} & \mathbf{0} \\ P & \mathbf{0} & -\gamma I_{2n_x} \end{bmatrix}.$$

Since $\gamma\beta$ is a positive constant, (19) is true if the following inequality is satisfied:

$$\dot{V}_t \leq \begin{bmatrix} E_t \\ \Phi_t \\ \varepsilon_t \end{bmatrix}^\top \Lambda \begin{bmatrix} E_t \\ \Phi_t \\ \varepsilon_t \end{bmatrix} - \alpha V_t + \gamma \|\varepsilon_t\|_2^2. \quad (20)$$

According to Sontag and Wang (1995), the system (17) is ISS if $\Lambda \preceq 0$ since $\|\varepsilon_t\|_2 < \infty$ by Assumptions 2.2 and 2.4. Then, if condition (15b) is satisfied, the framer (6) is a robust interval observer for (1) in the fault-free case.

In addition, since $\Lambda \preceq 0$, the quadratic term in the right hand side of inequality (20) is negative. Therefore, the following inequality is satisfied:

$$\dot{V}_t \leq -\alpha V_t + \gamma \|\varepsilon\|_\infty^2$$

or, integrating the differential inequality:

$$\begin{aligned} V_t &\leq V_0 e^{-\alpha t} + \gamma (1 - e^{-\alpha t}) \|\varepsilon\|_\infty^2 \\ &\leq V_0 e^{-\alpha t} + \gamma \|\varepsilon\|_\infty^2 \end{aligned} \quad (21)$$

since $e^{-\alpha t} \leq 1$. From (3) and (4), $R_t = CE_t$ (since $f_t = \mathbf{0}$). If:

$$\|R_t\|_2^2 \leq \mu \left(V_t + (\mu - \gamma) \|\varepsilon\|_\infty^2 \right) \quad (22)$$

then (16) is satisfied since V_t satisfies (21). Moreover, condition (22) is alternatively written:

$$\begin{bmatrix} P - C^\top C / \mu & \mathbf{0} \\ \mathbf{0} & \mu - \gamma \end{bmatrix} \succeq 0$$

which is equivalent, by Schur complement (Boyd et al., 1994), to condition (15c). \square

From Theorem 3.2, the matrices \underline{L} and \bar{L} are obtained as $\text{diag}(\underline{L}, \bar{L}) = P^{-1}Z$ by minimizing μ subject to the constraints (15a) to (15c).

3.3. Fault detection

The interval observer (6) tuned with the procedure described in Theorem 3.2 is used for fault detection. The structure of the fault detector is presented in Figure 1.

The fault-free output signal is injected into the interval observer (6). Then, based on equations (3) and (4), the residual generator provides the residual interval $[r_t, \bar{r}_t]$. Therefore, if $\mathbf{0} \notin [r_t, \bar{r}_t]$, the system is subject to a fault $f_t \neq \mathbf{0}$ and if $\mathbf{0} \in [r_t, \bar{r}_t]$, the system is fault-free or subject to a low-magnitude fault which cannot be detected by the proposed fault detection scheme.

Remark 3.1. The present paper focuses on the design of a TNL interval observer with L_∞ performance for robust sensor fault detection. The sensibility analysis of the proposed interval observer, characterized for example by the minimum detectable fault approach (Meseguer et al., 2010; Pourasghar et al., 2020), is therefore not addressed here and left to subsequent works.

4. Simulation results

To assess the efficiency of the proposed fault detection algorithm, the example of a damped mass-spring system (Scherer, 2012) is considered. This system obeys the LPV dynamics:

$$\begin{aligned} \dot{x}_t &= \begin{bmatrix} 0 & 1 \\ 2 + \rho_t & -1 \end{bmatrix} x_t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_t + w_t \\ y_t &= [1 \ 0] x_t + f_t \end{aligned}$$

where $x_t^\top = [p_t \ \dot{p}_t]$, with p_t the position of the mass, so that:

$$\begin{aligned} A_0 &= \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}, & \Delta A(\rho_t) &= \begin{bmatrix} 0 & 0 \\ \rho_t & 0 \end{bmatrix}, \\ B_0 &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, & \Delta B(\rho_t) &= \mathbf{0}, \end{aligned}$$

$D_0 = I_2$, $\Delta D(\rho_t) = \mathbf{0}$ and $C = [1 \ 0]$.

For this example, it is assumed that $\rho_t = \sin(0.3t)$, u_t is the square wave $u_t = \text{sgn}(\sin(t))$ and $w_t^\top = 0.1 [\cos(2t) \ \sin(3t)]$. Therefore, $\bar{w}_t = -\underline{w}_t = 0.1$ and:

$$\overline{\Delta A} = -\underline{\Delta A} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}.$$

The initial state vector is $x_0 = \mathbf{0}$ and the bounds for the initial state are $\bar{x}_0 = -\underline{x}_0 = 0.1 \cdot \mathbf{1}_2$.

The value of the matrix Ξ is a design parameter of the proposed interval observer. It can be chosen in different ways, such as to minimize the values of \underline{l}_ϕ and \bar{l}_ϕ . For the sake of simplicity, it is chosen here as:

$$\Xi = \begin{bmatrix} 0.1 & 0 & -0.1 \\ -3 & 0 & 3 \end{bmatrix}$$

- Boyd, S., L. El Ghaoui, E. Feron, and V. Balakrishnan (1994). *Linear Matrix Inequalities in System and Control Theory*. Society for Industrial and Applied Mathematics.
- Chebotarev, S., D. Efimov, T. Raïssi, and A. Zolghadri (2015). Interval observers for continuous-time LPV systems with L_1/L_2 performance. *Automatica* 58, 82–89.
- Combastel, C. (2005). A state bounding observer for uncertain non-linear continuous-time systems based on zonotopes. In *Proc. 44th IEEE Conf. Decis. Control*, pp. 7228–7234.
- Efimov, D., L. Fridman, T. Raïssi, A. Zolghadri, and R. Seydou (2012). Interval estimation for LPV systems applying high order sliding mode techniques. *Automatica* 48(9), 2365–2371.
- Efimov, D., T. Raïssi, and A. Zolghadri (2013). Control of nonlinear and LPV systems: interval observer-based framework. *IEEE Trans. Autom. Contr.* 58(3), 773–778.
- Garbouj, Y., T. N. Dinh, T. Raïssi, T. Zouari, and M. Ksouri (2020). Optimal interval observer for switched Takagi-Sugeno systems: an application to interval fault estimation. *IEEE Trans. Fuzzy Syst.*, to appear.
- Ifqir, S., D. Ichalal, N. Ait-Oufroukh, and S. Mammar (2020). Adaptive threshold generation for vehicle fault detection using switched T-S interval observers. *IEEE Trans. Ind. Electron.* 67(6), 5030–5040.
- Lamouchi, R., T. Raïssi, M. Amairi, and M. Aoun (2018). Interval observer framework for fault-tolerant control of linear parameter-varying systems. *Int. J. Control* 91(3), 524–533.
- Li, D., Y. Wang, J. Wang, C. Wang, and Y. Duan (2020). Recent advances in sensor fault diagnosis: A review. *Sens. Actuators A Phys.* 309, 111990.
- Li, J., Z. Wang, T. Raïssi, and Y. Shen (2020). Unknown input observer design for linear parameter-varying systems in a bounded error context. *IEEE Trans. Autom. Contr.*, to appear.
- Li, J., Z. Wang, W. Zhang, T. Raïssi, and Y. Shen (2019). Interval observer design for continuous-time linear parameter-varying systems. *Syst. Control Lett.* 134, 104541.
- Mazenc, F. and O. Bernard (2011). Interval observers for linear time-invariant systems with disturbances. *Automatica* 47(1), 140–147.
- Meseguer, J., V. Puig, T. Escobet, and J. Saludes (2010). Observer gain effect in linear interval observer-based fault detection. *J. Process Control* 20(8), 944–956.
- Nejjari, F., V. Puig, S. Montes de Oca, and A. Sadeghzadeh (2009). Robust fault detection for LPV systems using interval observers and zonotopes. In *Proc. 48th IEEE Conf. Decis. Control & 28th Chinese Control Conf.*, pp. 1002–1007. IEEE.
- Pourasghar, M., V. Puig, and C. Ocampo-Martinez (2020). Characterisation of interval-observer fault detection and isolation properties using the set-invariance approach. *J. Franklin Inst.* 357, 1853–1886.
- Raïssi, T., D. Efimov, and A. Zolghadri (2011). Interval state estimation for a class of nonlinear systems. *IEEE Trans. Autom. Control* 47(1), 140–147.
- Rao, C. R. and S. K. Mitra (1972). Generalized inverse of a matrix and its applications. In *Proc. 6th Berkeley Symp. on Math. Statist. and Prob.*, Volume 1, pp. 601–620.
- Robinson, E. I., J. Marzat, and T. Raïssi (2020). Prognosis of uncertain linear time-invariant discrete systems using unknown input interval observer. *Int. J. Control* 93, 2690–2706.
- Rotondo, D., A. Cristofaro, T. A. Johansen, F. Nejjari, and V. Puig (2018). State estimation and decoupling of unknown inputs in uncertain LPV systems using interval observers. *Int. J. Control* 91(8), 1944–1961.
- Rotondo, D., A. Cristofaro, T. A. Johansen, F. Nejjari, and V. Puig (2019). Robust fault and icing diagnosis in unmanned aerial vehicles using LPV interval observers. *Int. J. Robust Nonlinear Control* 29(16), 5456–5480.
- Scherer, C. W. (2012). A tutorial on the control of linear parameter-varying systems. In *Proc. Am. Control Conf.*
- Shamma, J. S. (2012). An overview of LPV systems. In J. Mohammadpour and C. W. Scherer (Eds.), *Control of Linear Parameter Varying Systems with Applications*, pp. 3–26. Springer.
- Sontag, E. D. and Y. Wang (1995). On characterizations of the input-to-state stability property. *Syst. Control Lett.* 24, 351–359.
- Varga, A. (2017). *Solving Fault Diagnosis Problems*. Springer.
- Wan, Y., V. Puig, C. Ocampo-Martinez, Y. Wang, E. Harinath, and R. D. Braatz (2020). Fault detection for uncertain LPV systems using probabilistic set-membership parity relation. *J. Process Control* 87, 27–36.
- Wang, Y., D. M. Bevly, and R. Rajamani (2015). Interval observer design for LPV systems with parametric uncertainty. *Automatica* 60, 79–85.
- Wang, Z., C.-C. Lim, and Y. Shen (2018). Interval observer design for uncertain discrete-time linear systems. *Syst. Control Lett.* 116, 41–46.
- Zammali, C., J. Van Gorp, Z. Wang, and T. Raïssi (2021). Sensor fault detection for switched systems using interval observer with L_∞ performance. *Eur. J. Control* 57, 147–156.
- Zheng, G., D. Efimov, and W. Perruquetti (2016). Design of interval observer for a class of uncertain unobservable nonlinear systems. *Automatica* 63, 167–174.