Interval Observer Design for Nonlinear Systems using Simplified Contraction theory

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ORIGINAL RESEARCH

Interval observer design for nonlinear systems using simplified contraction theory

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Abstract

In this work, vector framework based contraction theory to design interval observers for a class of nonlinear systems having disturbances, inputs and outputs is exploited. The main feature is that it does not require the formulation of error dynamics to show the convergence properties and need not require the construction of a Lyapunov candidate function without any idea of the structure of the function. Specifically, it performs the convergence analysis through a comparison system which has specified properties. Furthermore, this theory is exploited to design dynamic output feedback control through the obtained state bounds from the constructed interval observer and the system outputs, to make the interval observer to be globally asymptotically stable. Examples with simulation outcomes are provided to validate the theoretical results.

1 | INTRODUCTION

For the purpose of feedback control, all the states of a dynamical system are not measurable due to a fewer number of sensors. Thus, to reproduce all the information of these systems, that is, to estimate the unmeasured states, observers [1] are used. Basically, there is no need of state information in transient periods for the problems of stabilisation and tracking. However, in the case of monitoring purposes and other essential practical applications, there is a great need of state estimation at all times. This type of knowledge of state estimation at all times cannot be available from classical observers like Luenberger observer [2]. Moreover, classical observers do not provide guaranteed exact estimates in the presence of perturbations and uncertainties. To overcome these shortcomings and provide guaranteed state estimations at all times in the presence of perturbations and uncertainties, there comes the notion of interval observers [3]. They play an important role in fulfilling the practical demands. In fact, they have a major advantage over classical observers in the sense that they can easily cope up with large uncertainties and have been successfully applied in biological fields [4] where large uncertainties are present. Moreover, in the recent years, interval observers have also been applied for several industrial and engineering applications [5–9]. The structure of interval observer comprises of two dynamical equations that estimate upper and lower bounds (interval) of the state vector at all times when the initial conditions are unknown and there are uncertainties present in the system, however, these initial conditions and uncertainties are bounded.

In the analysis and design of interval observers, two important conditions need to be satisfied. The first is the framer property, and the other one is the convergence property. To prove the framer property, one needs to have a positive dynamical system, that is, for example, for the linear systems case, the system matrix should be a Metzler matrix. This property is necessary to ensure the partial ordering of the state trajectories. In other words, if the initial conditions are bounded between two values (known), then the real state trajectory is bounded by

the trajectories originating from these bounds. Furthermore, if the system is a non-positive system, there are various methods available in the literature for linear systems in particular [10], in which by using time-varying transformations, the non-positive system gets transformed into a positive system. In ref. [11], an internal positive realisation scheme has been used to design interval observers. Moreover, several works were reported in the literature for the design of interval observers for nonlinear systems and discrete-time systems as well (see refs. [12, 14–16]). The second important property for the design of interval observers is the convergence property. One needs this convergence property to show that the norm of the error between the state bounds converges to zero when the disturbance acting on the system is zero. Several researchers (see refs. [17–20]) proved this convergence property using Lyapunov stability analysis.

But still, in these types of convergence problems, there is a need to find the convergence of the estimated state bounds and the real state trajectory with respect to each other rather than their convergence to a particular point or set. To show convergence of trajectories with respect to each other, a particularly stronger theory than Lyapunov stability, known as contraction theory [21] exists in the literature. This theory differs from Lyapunov stability in the sense that it does not require a specific attractor to show stability and does not require the construction of a Lyapunov candidate function without having any knowledge of its structure. This theory analyses the stability in terms of a differential framework, that is, a system is said to be stable in some region if its final behaviour is independent of initial conditions and temporary perturbations, and hence, state trajectories converge with respect to one another, therefore, this form of stability is called as convergence, to remove any ambiguity.

However, contraction analysis usually utilised in the literature requires a complex procedure to solve matrix measure or largest eigenvalue of the symmetric part of the Jacobian of large-scale nonlinear systems to show convergence, since it considers a scalar distance function. In such cases, convergence is performed through a recently developed theory, a vector based contraction theory [22]. This theory has been exploited for the design of controller and observer in ref. [24]. It utilises vector valued distances (vector valued norm) and a comparison system to show convergence analysis. It provides an advantage over the Lyapunov stability approach, in the sense that it has well-specified structure of the comparison system to prove convergence.

In this work, an interval observer for nonlinear systems considering inputs, outputs, and disturbances is designed using a vector framework based contraction theory. In fact, this theory is utilised to prove the convergence property of the interval observer through a comparison system with specified properties. It provides the advantage over other existing approaches in the way that it does not require the formulation of error dynamics and need not require the Lyapunov candidate function to show convergence. In addition, this theory is exploited to design dynamic feedback control by the bounds obtained from the constructed interval observer and the system outputs to make the interval observer to be globally asymptotically stable. In the end, several examples with simulation outcomes are illustrated to validate the developed results. It can be observed that many practical systems such as TORA (translational oscillator rotating actuator) [25] and electromechanical system [27] belong to the family of systems affine in the unmeasured part of the state variables. This fact motivates us to study this class of nonlinear systems in the present work. To the author's best knowledge, an interval observer has not been designed yet using a contraction framework particularly vector based contraction.

The further part of the paper is framed as follows. The notions and preliminaries are entitled in Section 2. Section 3 provides the main results. Examples with respective simulations are presented in Section 4. Finally, the conclusions end the paper.

2 | NOTATIONS AND PRELIMINARIES

Let $\mathbb{R}$ and $\mathbb{R}_+$ denote the set of real numbers and non-negative real numbers. The $n-$tuple vector is denoted by $\mathbb{R}^n$. A matrix is said to be a Metzler matrix if its off-diagonal entries are non-negative. $\|\cdot\|$ is known as the Euclidean norm. $B \geq 0$ means that its entries are non-negative (i.e. $b_{ij} \geq 0, i \neq j$), where B is a real matrix. We define vector valued norm of $\chi \in \mathbb{R}^n$ as $\|\chi\|_{\mathcal{V}}$ in Equation (4). We use component-wise inequality, that is, $p < q$ or $p \leq q$, for $p = [p_1, p_2, \dots, p_n]^\top$ and $q = [q_1, q_2, \dots, q_n]^\top$, if $p_i < q_i$ or $p_i \leq q_i$ for each $i = 1, 2, \dots, n$. The diagonal matrix for a vector $r = [r_1, r_2, \dots, r_n]^\top \in \mathbb{R}^n$ is represented by $\mathrm{diag}(r)$. A vector $d = [d_1, d_2, \dots, d_n]^\top \in \mathbb{R}^n$ can also be written as $\mathrm{diag}(d)\mathbb{1}$, where $\mathbb{1}$ is the $n-$tuple column vector: $[1, 1, \dots, 1]^\top$. The set of all continuous functions from L to $\mathcal{Q}$ is denoted by $C[L, \mathcal{Q}]$, where L and $\mathcal{Q}$ are non-empty sets. Assume $M \subset \mathbb{R}^n$ and $v = (v_1, v_2, \dots, v_n)^\top$ be the element of M, we say $\psi = (\psi_1, \psi_2, \dots, \psi_n)^\top \in C[M, \mathbb{R}^n]$ as quasi-monotone non-decreasing function on M if ψ_i is non-decreasing in $v_j \ \forall \ i = 1, 2, \dots, j-1, j+1, \dots, n$ and $j \in \{1, 2, \dots, n\}$ [23].

2.1 | Contraction analysis

Consider a nonlinear system

$$\Pi : \ \dot{\chi} = \mathbb{F}(t, \chi), \ \ \chi(t_0) = \chi_0, \ t_0 \geq 0, \tag{1}$$

where $\mathbb{F} : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ is a smooth nonlinear field and $\chi \in \mathbb{R}^n$ represents the state vector. Let $\phi(t, \chi_0)$ be the unique solution of Π. Taking the partial differentiation of the system Π at fixed time t,

$$\delta\dot{\chi} = \frac{\partial \mathbb{F}(t, \chi)}{\partial \chi}\delta\chi + \frac{\partial \mathbb{F}(t, \chi)}{\partial t}\delta t$$

Since time t is fixed, then $\delta t = 0$, hence the virtual dynamics of the system Π at fixed time t is

$$\Pi_\delta : \ \delta\dot{\chi} = \frac{\partial \mathbb{F}(t, \phi(t, \chi_0))}{\partial \chi}\delta\chi = \mathcal{H}(t, \delta\chi, \chi), \tag{2}$$

where $\mathcal{H} : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a general nonlinear function and $\delta z \in \mathbb{R}^n$ is the infinitesimal virtual distance between any pair of trajectories of system Π. We denote $\delta\phi(t, z_0, \delta z_0)$ as the solution to Equation (2) with initial condition δz_0 along $\phi(t, z_0)$. Thus, the derivative of scalar virtual distance $\delta z^\top \delta z$ is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\delta z^\top \delta z\right) = 2\delta z^\top \delta \dot{z} = 2\delta z^\top \frac{\partial \mathbb{F}}{\partial z}\delta z. \tag{3}$$

Then, any virtual distance $\|\delta z\|$ converges exponentially to zero as $t \to \infty$, if $\sigma_{\max}(t, z)$ is uniformly negative definite (see ref. [21]), where $\sigma_{\max}(t, z)$ is the largest eigenvalue of the symmetric part of the Jacobian $\frac{\partial \mathbb{F}}{\partial z}$ matrix. Or in simple words, this virtual distance converges exponentially to zero, if the Jacobian matrix is a Hurwitz matrix. Thus, a system (1) is said to be a contracting (convergent) system.

2.2 | Vector based contraction analysis

We note that it is quite difficult to formulate the Lyapunov candidate function without having any knowledge of its structure to prove stability. Moreover, the calculation of the largest eigenvalue of the symmetric part of the Jacobian matrix is quite complex for large-scale nonlinear systems in the case of contraction analysis. In such cases, convergence analysis is done through comparison systems. We discuss vector based contraction theory as follows. In this, vector valued distance $\|\cdot\|_{\mathcal{V}} : \mathbb{R}^n \to \mathbb{R}^n$ is defined by

$$\|\delta z\|_{\mathcal{V}} = \sqrt{L\,(\mathrm{diag}(\delta z))^2 \mathbb{1}}, \tag{4}$$

where $L = [l_{ij}]_{n\times n}$ is a non-zero matrix with all l_{ij} non-negative. Note that for $\delta z = (\delta z_1, \delta z_2, \dots, \delta z_n)^\top \in \mathbb{R}^n$,

$$(\mathrm{diag}(\delta z))^2 = \begin{bmatrix} \delta z_1^2 & 0 & \cdots & 0 \\ 0 & \delta z_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \delta z_n^2 \end{bmatrix}, (\mathrm{diag}(\delta z))^2 \mathbb{1} = \begin{bmatrix} \delta z_1^2 \\ \delta z_2^2 \\ \vdots \\ \delta z_n^2 \end{bmatrix}$$

Thus, explicitly, Equation (4) can be written as

$$\|\delta z\|_{\mathcal{V}} = \begin{bmatrix} D_1(\delta z) \\ D_2(\delta z) \\ \vdots \\ D_n(\delta z) \end{bmatrix} := \begin{bmatrix} \sqrt{\sum_{j=1}^n l_{1j}\delta z_j^2} \\ \sqrt{\sum_{j=1}^n l_{2j}\delta z_j^2} \\ \vdots \\ \sqrt{\sum_{j=1}^n l_{nj}\delta z_j^2} \end{bmatrix}.$$

The main result of this theory is given as follows.

Lemma 1 [22]. *Consider the system (1). Suppose there exists solution $\delta\phi(t, z_0, \delta z_0)$ of the system (2) such that*

$$\frac{d}{dt}\|\delta z\|_{\mathcal{V}}^2 < \psi\left(t, L(diag(\delta\phi))^2\mathbb{1}, \phi\right)$$

where $L = [l_{ij}]_{n\times n}$ with $l_{ij} \geq 0$ for all $i, j = 1, 2, \dots, n$ and $\psi \in C[\mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n, \mathbb{R}^n]$, $(t, w, z) \to \psi(t, w, z)$ is a quasi-monotone non-decreasing function in $w \in \mathbb{R}^n$. Further, if there exists a unique solution $Z(t)$ of the following comparison system

$$\dot{w} = \psi(t, w, z),\ w(t_0) = w_0 \geq 0. \tag{5}$$

Then, any solution $\delta\phi(t, z_0, \delta z_0)$ of (2) on $t \geq t_0$ satisfying $L\,(diag(\delta z_0))^2\mathbb{1} < w_0$ follows

$$L\,(diag(\delta\phi(t)))^2\mathbb{1} < Z(t) \text{ for all } t \geq t_0.$$

From above, it is concluded that

If $Z(t) \to 0$ as $t \to \infty$, then $\|\delta\phi(t)\| \to 0$ as $t \to \infty$, which implies that all solutions of the original system (1) converge to each other as $t \to \infty$.

Remark 1. It is important to discuss that how to design the comparison system (5). Let us see some examples. In most of the cases, this system (5) can be intuitively selected as a linear system (let us say $\dot{w} = Aw$) in such a way that the matrix A has non-negative off-diagonal entries (follows the property of quasi-monotone non-decreasing). To further prove that the solution of the comparison system $Z(t) \to 0$ as $t \to \infty$ which implies that $\|\delta\phi(t)\| \to 0$ as $t \to \infty$ (as seen in the conclusion of the above result), the A matrix should be a Hurwitz matrix to prove the comparison system to be contracting. In some of the cases, the system (5) can be selected as a nonlinear system, then for $1 \leq i \leq n$ and all $1 \leq j \leq n, j \neq i, p_i = q_i, p_j \leq q_j$ implies that $\psi_i(t, p, z) \leq \psi_i(t, q, z)$ for the ith component of $\psi(t, \cdot)$ and for each t and z to prove the function ψ to be quasi-monotone non-decreasing.

2.3 | General conditions in interval observer design

Consider a general nonlinear system

$$\dot{z} = f(t, z, d(t)), \quad z(t_0) = z_0, \quad y = g(z) \tag{6}$$

where $z \in \mathbb{R}^n$ is a state vector, $y \in \mathbb{R}^p$ is the system output, $d(t) \in \mathbb{R}^l$ is a locally Lipschitz bounded disturbance, that is, $\underline{d}(t) \leq d(t) \leq \overline{d}(t)$, $\underline{d}(t) \in \mathbb{R}^l$, $\overline{d}(t) \in \mathbb{R}^l$, $f : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^l \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^p$ are locally Lipschitz nonlinear functions. The initial condition is unknown but bounded between two values, $\underline{z}(t_0) \leq z(t_0) \leq \overline{z}(t_0)$.

Now, if the following system

$$\dot{\zeta} = h(t, \zeta(t), y(t), d'(t)) \tag{7}$$

with initial condition $\zeta_0 = G(t_0, \overline{z}_0, \underline{z}_0)$, $d' = (\overline{d}, \underline{d})$ and bounds for z: $\overline{z} = \overline{H}(t, \zeta)$, $\underline{z} = \underline{H}(t, \zeta)$ (where $h, G, \overline{H}, \underline{H}$ are Lipschitz functions with appropriate dimensions), $\zeta \in \mathbb{R}^n$, follows two conditions:

- *Framer property*: For any set of initial conditions $z_0, \underline{z}_0$ and $\overline{z}_0$ in $\mathbb{R}^n$ satisfying $\underline{z}_0 \leq z_0 \leq \overline{z}_0$, the solutions of system (6) and (7) follow

$$\underline{z}(t) = \underline{H}(t, \zeta(t)) \leq z(t) \leq \overline{z}(t) = \overline{H}(t, \zeta(t)), \quad \forall\, t \geq t_0$$

  with initial conditions z_0 and $G(t_0, \overline{z}_0, \underline{z}_0)$.
- *Convergence property*: when $d(t) = 0$, then the norm of the error $\|e(t)\| = \|\overline{z}(t) - z(t)\|$ or $\|z(t) - \underline{z}(t)\|$ converges exponentially to zero.

  Then, it is called an interval observer for the system (6).

3 | MAIN RESULTS

This section provides the main results. We consider following class of nonlinear systems for our development.

$$\begin{aligned} \dot{z} &= A(y)z + f(y, u) + Bd(t) \\ y &= Cz \end{aligned} \tag{8}$$

where $z \in \mathbb{R}^n$ represents the states, $u \in \mathbb{R}^q$ is the control input, $y \in \mathbb{R}^p$ is the system output, f is a general smooth nonlinear function, $d(t) \in \mathbb{R}^l$ is the unknown bounded disturbance with $\underline{d}(t) \leq d(t) \leq \overline{d}(t)$ and initial conditions are also unknown but bounded between two bounds, that is, $\underline{z}_0 \leq z_0 \leq \overline{z}_0$. $A(y)$, B and C are the matrices with appropriate dimensions with $A(y)$ as a Metzler matrix for all $y \in \mathbb{R}^p$, $B \geq 0$ and $C \geq 0$. Let $z(t)$ be the unique solution of system (8).

Lemma 2 (Framer design). *Consider two dynamics*

$$\dot{\overline{z}} = A(y)\overline{z} + K(y - \overline{y}) + f(y, u) + B\overline{d}, \quad \overline{z}(t_0) = \overline{z}_0 \tag{9}$$

$$\dot{\underline{z}} = A(y)\underline{z} + K(y - \underline{y}) + f(y, u) + B\underline{d}, \quad \underline{z}(t_0) = \underline{z}_0 \tag{10}$$

where $\overline{y} = C\overline{z}$, $\underline{y} = C\underline{z}$ *and* K *is a gain matrix with appropriate dimension such that* $-KC \geq 0$. *Let* $\overline{z}(t)$ *and* $\underline{z}(t)$ *be the unique solutions of the systems (9) and (10) respectively, then* $\overline{z}(t)$ *and* $\underline{z}(t)$ *are the upper and lower bounds for the state* $z(t)$.

Proof. Let $\overline{e} = \overline{z} - z$ and $\underline{e} = z - \underline{z}$ be the upper observation and lower observation errors, respectively. The aim is to prove that $\overline{e}(t)$ and $\underline{e}(t)$ are non-negative. The dynamics of the upper error follow

$$\dot{\overline{e}} = (A(y) - KC)\overline{e} + B(\overline{d} - d). \tag{11}$$

Similarly, the dynamics of the lower error are described by

$$\dot{\underline{e}} = (A(y) - KC)\underline{e} + B(d - \underline{d}). \tag{12}$$

Because $\underline{d} \leq d \leq \overline{d}$ and $B \geq 0$, we have $B(\overline{d} - d) \geq 0$ and $B(d - \underline{d}) \geq 0$. Bearing in mind $A(y)$ Metzler for all $y \in \mathbb{R}^p$ and $-KC \geq 0$, we deduce that $A(y) - KC$ Metzler for all $y \in \mathbb{R}^p$. Moreover, from the fact that $\overline{e}_0 = \overline{z}_0 - z_0 \geq 0$ and $\underline{e}_0 = z_0 - \underline{z}_0 \geq 0$, it follows that, for all $t \geq 0$, $\overline{e}(t) \geq 0$ and $\underline{e}(t) \geq 0$. Thus, $\underline{z}(t) \leq z(t) \leq \overline{z}(t)$. This allows us to conclude that Equations (9) and (10) is a framer for Equation (8). □

Remark 2. If $A(y)$ is not a Metzler matrix, we can use a change of coordinates or any other transformations existing in the literature to make it Metzler as shown in particular in refs. [10, 12, 13]. We illustrate the case where $A(y)$ is not a Metzler matrix in Example 4 of Section 4.

Usually, it is not very difficult to achieve the framer property, which is the notion of providing intervals in which state variables stay, if one does not care the length of estimated intervals. In fact, one can use an artificial system which over-bounds terms to secure the positivity (non-negativity, more precisely) ensuring the framer property [28]. Therefore, tools and component ideas of the framer design are not necessarily novel. The framer design proposed in Lemma 2 is the same as in ref. [12] and as in many other studies on interval observers for nonlinear systems in the literature. The goal of this paper is to present a reasonable convergence property of the interval, which is not always clear. In Assumption 3 of ref. [12], the authors clarify how they can qualify the convergence property by employing Lyapunov stability, which requires complex and difficult computations, as the authors illustrated them in the numerical example. The approach we propose in the present paper is very different from ref. [12] since it is based on ideas from contraction theory, and hardly any computation is needed to expose it.

We introduce the following theorem as guidelines for selecting K in Lemma 2 to make framer Equations (9) and (10) become interval observer (i.e. framer satisfies a convergence property) for Equation (8).

Theorem 1. *Consider the system (8) without disturbance* $d(t)$. *Let us suppose that there exists the gain matrix* K *in Lemma 2 such that the vector valued distance derivative along virtual dynamics of the system (9) follows*

$$\frac{d}{dt}\|\delta\overline{z}\|_{\mathcal{V}}^2 < \psi\left(L\,(diag(\delta\overline{z}))^2\mathbb{1}\right), \quad \forall\, t \geq t_0$$

for a non-zero matrix $L = [l_{ij}]_{n\times n}$ *with all real* $l_{ij} \geq 0$, *function* $\psi \in C[\mathbb{R}^n, \mathbb{R}^n]$ *satisfies the quasi-monotonicity non-decreasing property, and*

the comparison system obtained from this inequality is contracting, then the error $\|e(t)\| = \|\overline{z}(t) - z(t)\|$ *converges exponentially to zero, that is, for some constants* $s > 0$ *and* $k > 0$

$$\|e(t)\| \leq ke^{s(t-t_0)}\|e(t_0)\|.$$

Proof. Consider the vector valued distance function defined by Equation (4). The vector valued distance derivative along virtual dynamics of the system (9) is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}(\|\delta\overline{z}\|_{\mathcal{V}}^2) = 2L\ \mathrm{diag}(\delta\overline{z})\ (\mathrm{diag}\dot{\delta\overline{z}})\mathbb{1}$$

The gain matrix K is designed such that the above equality transforms to the following inequality

$$\frac{\mathrm{d}}{\mathrm{d}t}(\|\delta\overline{z}\|_{\mathcal{V}}^2) < \psi\left(L\ (\mathrm{diag}(\delta\overline{z}))^2\mathbb{1}\right) \qquad \forall\, t \geq t_0, \tag{13}$$

where ψ is a quasi-monotone non-decreasing function and the comparison system obtained from this inequality (13), let's say, $\dot{w} = \psi(w), w \in \mathbb{R}^n$ is contracting. Hence, from Lemma 1, the distance between any pair of trajectories $\|\delta\overline{z}\|$ of the estimated system (9) converges exponentially to zero since the comparison system trajectories converge exponentially to zero (comparison system is contracting). This means that, for some constants $k > 0$ and $s > 0$, we have

$$\|\overline{z}_1(t) - \overline{z}_2(t)\| \leq\ ke^{-s(t-t_0)}\|\overline{z}_1(t_0) - \overline{z}_2(t_0)\|,\ \ \forall\, t \geq t_0, \tag{14}$$

for any two solutions $\overline{z}_1(t)$ and $\overline{z}_2(t)$ of the system (9). Furthermore, when $d(t) = 0$, $z(t)$ is a particular solution of the system (9) since system (9) and Equation (8) only differ in the correction term. Hence, the correction term vanishes when $z(t)$ is the solution of Equations (9). Thus, $\overline{z}_2(t)$ and $\overline{z}_1(t)$ can be replaced by $z(t)$ and $\overline{z}(t)$ respectively, therefore the above Equation (14) becomes

$$\|e(t)\| = \|\overline{z}(t) - z(t)\| \leq ke^{-s(t-t_0)}\|e(t_0)\|.$$

Hence, it is proved that the error of estimation converges exponentially to zero. This completes the proof.

In a similar way, one can prove the exponential convergence of the error $\|e(t)\| = \|z(t) - \underline{z}(t)\|$ to zero. The proposed method has the advantage that it does not require the error dynamics formulation to show that the estimation error $\|e(t)\| = \|\overline{z}(t) - z(t)\| = \|z(t) - \underline{z}(t)\|$ converges exponentially to zero, when the disturbance $\overline{d}(t) = 0$. It only requires the virtual dynamics of the estimation dynamics (9) and (10) to show that the derivative of the squared vector distance along this virtual dynamics follows Equation (13).

Now, we provide a result to design feedback control $u(y, \overline{z})$ to make the interval observer (9) and (10) to be asymptotic stable when d, $\overline{d}$ and $\underline{d}$ are zero. □

Theorem 2. *Consider the system (8), (9), (10) with* $d(t) = 0$, $\overline{d}$ *and* $\underline{d} = 0$. *Let us assume that the origin is the equilibrium point of system (8). Suppose there exists the control* $u(y, z)$, $u(0, 0) = 0$, u *as a general smooth nonlinear function, such that the derivative of vector valued distance along virtual dynamics of the system (8) follows*

$$\frac{d}{dt}(\|\delta z\|_{\mathcal{V}}^2) < \psi\left(L\ (diag(\delta z))^2\mathbb{1}, z\right) \qquad \forall\, t \geq t_0,$$

for $L = [l_{ij}]_{n\times n}$, $l_{ij} \geq 0$, $\psi \in C[\mathbb{R}^n \times \mathbb{R}^n,\ \mathbb{R}^n]$ *satisfies the property of quasi-monotonicity non-decreasing, and the comparison system obtained from this inequality is contracting. Then, the origin of the system (8) is globally asymptotically stable. Further, an interval observer (9) and (10) is asymptotically stable with the control* $u(y, \overline{z})$, *when* $\overline{d}$ *and* $\underline{d}$ *are zero.*

Proof. The virtual dynamics of the system (8) with the control $u(y, z)$ at fixed time t is obtained as

$$\delta\dot{z} = \left(\frac{\partial A(y)}{\partial z}z + A(y) + \frac{\partial f}{\partial z} + \frac{\partial f}{\partial u}\frac{\partial u}{\partial z}\right)\delta z \tag{15}$$

Consider the vector valued distance function defined by Equation (4). The vector valued distance derivative along the trajectories of Equation (15) is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}(\|\delta z\|_{\mathcal{V}}^2) = 2L\ \mathrm{diag}(\delta z)\ (\mathrm{diag}(\frac{\partial A(y)}{\partial z}z + A(y) + \frac{\partial f}{\partial z} + \frac{\partial f}{\partial u}\frac{\partial u}{\partial z})\delta z)\mathbb{1}$$

Now, the control $u(y, z)$ is designed such that the above vector valued distance derivative follows the inequality

$$\frac{\mathrm{d}}{\mathrm{d}t}(\|\delta z\|_{\mathcal{V}}^2) < \psi\left(L\ (\mathrm{diag}(\delta z))^2\mathbb{1}, z\right) \qquad \forall\, t \geq t_0,$$

where ψ is a quasi-monotone non-decreasing function and the comparison system obtained from this inequality, let us say, $\dot{w} = \psi(w, z), w \in \mathbb{R}^n$ is contracting. Thus, from the results of Lemma 1, the distance between any pair of trajectories $\|\delta z\|$ of the system (8) converges exponentially to zero, that is, the trajectories converge indeed to an equilibrium point (origin). Hence, it is proved that with the control $u(y, z)$, the origin of the system (8) is globally asymptotically stable.

Now, the proof goes in a similar way to show that the interval observer (9) and (10) is also asymptotically stable with the control $u(y, \overline{z})$, when $\overline{d}$ and $\underline{d}$ are zero.

In general, one can design $u(y, z)$ (respectively $u(y, \overline{z})$) using backstepping technique [29] and vector based contraction theory. In fact, each component of vector valued distance is utilised to prove the asymptotic stability of each subsystem. This control design procedure is clearly illustrated in Example 4 of Section 4. □

4 | SIMULATION EXAMPLES

Example 1. *Consider the system*

$$\dot{z}_1 = \sigma(z_2 + z_1) + d(t)$$

$$\dot{z}_2 = -\eta z_2 + \sin^2(z_1)z_3 + z_1^2 \sin^2(z_1 - z_3) + d(t) \quad (16)$$

$$\dot{z}_3 = \beta z_3 + \cos^2(z_1 - z_3)z_2 + \rho z_1 + d(t)$$

with outputs $y_1 = z_1$ *and* $y_2 = z_1 - z_3$, *where* $z = [z_1, z_2, z_3]^\top \in \mathbb{R}^3$ *is the state vector,* $d(t)$ *is the unknown disturbance with* $\underline{d} \le d(t) \le \overline{d}$. *Parameter values are given as* $\sigma = 1, \eta = 5, \rho = 5$ *and* $\beta = 5$.

We write the above system in the form

$$\dot{z} = A(y) + f(y) + Bd(t) \quad (17)$$

with

$$A(y) = \begin{bmatrix} \sigma & \sigma & 0 \\ 0 & -\eta & \sin^2(y_1) \\ 0 & \cos^2(y_2) & \beta \end{bmatrix}, \; f(y) = \begin{bmatrix} 0 \\ y_1^2 \sin^2(y_2) \\ \rho y_1 \end{bmatrix}$$

and $B = [1, 1, 1]^\top$. *It can be observed that* $A(y)$ *is Metzler for all* y_1 *and* y_2, *since* $\cos^2(y_2)$ *and* $\sin^2(y_1)$ *are always non-negative for all* y_1 *and* y_2, *but it is not a Hurwitz matrix. Now, an interval observer is designed as follows*

$$\dot{\overline{z}} = A(y)\overline{z} + K(y - \overline{y}) + f(y) + B\overline{d} \quad (18)$$

$$\dot{\underline{z}} = A(y)\underline{z} + K(y - \underline{y}) + f(y) + B\underline{d} \quad (19)$$

where $K = \begin{bmatrix} K_1 & 0 \\ 0 & 0 \\ 0 & -K_2 \end{bmatrix}$ *is the gain matrix and* $y = Cz$, $C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$. *We use vector based contraction approach with* $d(t) = 0$ *to prove the convergence property of the designed interval observer as the construction of the Lyapunov candidate is not easy for a nonlinear system and also there is no need of any specific attractor to prove convergence. Firstly, we consider the system (18), the virtual dynamics of the system (18) is given by*

$$\delta\dot{\overline{z}}_1 = (\sigma - K_1)\delta\overline{z}_1 + \sigma\delta\overline{z}_2$$

$$\delta\dot{\overline{z}}_2 = -\eta\delta\overline{z}_2 + \sin^2(y_1)\delta\overline{z}_3 \quad (20)$$

$$\delta\dot{\overline{z}}_3 = K_2\delta\overline{z}_1 + \cos^2(y_2)\delta\overline{z}_2 + (\beta - K_2)\delta\overline{z}_3$$

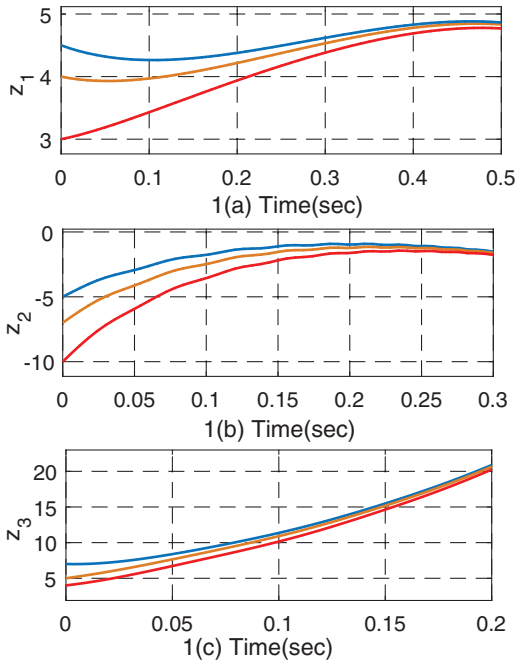

FIGURE 1 Interval observer for system (16) (1(a)(b)(c) with disturbance)

The derivative of the vector valued distance (assuming L *as* $\mathrm{diag}(\mathbb{1})$*) along the trajectories of Equation (20) is given by*

$$\frac{d}{dt}(\delta\overline{z}_1)^2 = 2(\sigma - K_1)\delta\overline{z}_1^2 + 2\sigma\delta\overline{z}_1\delta\overline{z}_2$$

$$\le (3\sigma - 2K_1)\delta\overline{z}_1^2 + \sigma\delta\overline{z}_2^2$$

$$\frac{d}{dt}(\delta\overline{z}_2)^2 \le (-2\eta + |\sin^2(y_1)|)\delta\overline{z}_2^2 + |\sin^2(y_1)|\delta\overline{z}_3^2$$

$$\frac{d}{dt}(\delta\overline{z}_3)^2 \le |K_2|\delta\overline{z}_1^2 + |\cos^2(y_2)|\delta\overline{z}_2^2 + (2\beta - 2K_2 + |K_2| + |\cos^2(y_2)|)\delta\overline{z}_3^2$$

From the above inequalities, we obtain the quasi-monotone non-decreasing (off-diagonal entries non-negative) comparison system, $\dot{w} = Gw$, *with*

$$G = \begin{bmatrix} (3\sigma - 2K_1) & \sigma & 0 \\ 0 & (|\sin^2(y_1)| - 2\eta) & |\sin^2(y_1)| \\ |K_2| & |\cos^2(y_2)| & \mathcal{Q} \end{bmatrix}$$

where $\mathcal{Q} = (2\beta - 2K_2 + |K_2| + |\cos^2(y_2)|)$. *We select the gains* $K_1 = 10$ *and* $K_2 = 40$ *to make the above matrix* G *Hurwitz (contracting) and* $-KC \ge 0$. *Hence, the original dynamics (18) is contracting. In a similar way, the dynamics (19) can be proved to be contracting. Thus, the system (18) and (19) is an interval observer for the system (16). The simulation results are shown in Figures 1 and 2 with and without disturbance respectively with values:* $3 \le z_1(0) \le 4.5$, $-10 \le z_2(0) \le -5$, $4 \le$

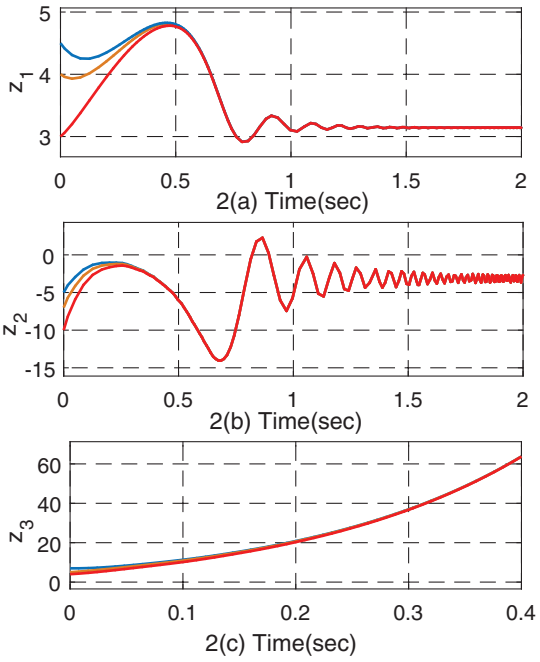

FIGURE 2 Interval observer for system (16) (2(a)(b)(c) without disturbance)

$z_3(0) \leq 7$, $z(0) = [4, -7, 5]^\top$, $d(t) = 0.25\sin(t)$, $\underline{d} = -0.25$ *and* $\overline{d} = 0.25$.

Example 2. *Consider the following self-excited nonlinear oscillator ([26])*

$$\begin{aligned}\dot{z}_1 &= z_2 + d(t)\\ \dot{z}_2 &= -\omega_1^2 \sin(z_1) - \rho z_2 + k_1 \arctan(k_2(z_1 - z_3)) + d(t)\\ \dot{z}_3 &= \omega_2(z_1 - z_3) + d(t)\end{aligned} \tag{21}$$

with outputs $y_1 = z_1$ *and* $y_2 = z_1 - z_3$, *where* $z = [z_1, z_2, z_3]^\top \in \mathbb{R}^3$ *is the state vector,* $d(t)$ *is the unknown disturbance with* $\underline{d} \leq d(t) \leq \overline{d}$. *Parameter values are given as:* $\omega_1 = \omega_2 = 40, k_1 = 5, k_2 = 10$ *and* $\rho = 1$.

We write the above system (21) in the form

$$\dot{z} = Az + f(y) + Bd(t) \tag{22}$$

with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\rho & 0 \\ \omega_2 & 0 & -\omega_2 \end{bmatrix},\ f(y) = \begin{bmatrix} 0 \\ -\omega_1^2 \sin(y_1) + k_1 \arctan(k_2 y_2) \\ 0 \end{bmatrix}$$

and $B = [1, 1, 1]^\top$. *It is observed that the system matrix* A *is Metzler but not Hurwitz matrix. Now, we design an interval observer as follows:*

$$\dot{\overline{z}} = A\overline{z} + K(y - \overline{y}) + f(y) + B\overline{d} \tag{23}$$

$$\dot{\underline{z}} = A\underline{z} + K(y - \underline{y}) + f(y) + B\underline{d} \tag{24}$$

where $K = \begin{bmatrix} K_1 & 0 \\ 0 & 0 \\ 0 & -K_2 \end{bmatrix}$ *is the gain matrix and* $y = Cz$, $C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$. *In a similar way, we consider first the system (23), the virtual dynamics of the system (23) is given by*

$$\begin{aligned}\delta\dot{\overline{z}}_1 &= -K_1\delta\overline{z}_1 + \delta\overline{z}_2\\ \delta\dot{\overline{z}}_2 &= -\rho\delta\overline{z}_2\\ \delta\dot{\overline{z}}_3 &= (\omega_2 + K_2)\delta\overline{z}_1 - (\omega_2 + K_2)\delta\overline{z}_3\end{aligned} \tag{25}$$

The derivative of the vector valued distance (assuming L *as* $\mathrm{diag}(\mathbb{1})$*) along the trajectories of (25) is given by*

$$\begin{aligned}\frac{d}{dt}(\delta\overline{z}_1)^2 &= -2K_1\delta\overline{z}_1^2 + 2\delta\overline{z}_1\delta\overline{z}_2\\ &\leq (1 - 2K_1)\delta\overline{z}_1^2 + \delta\overline{z}_2^2\\ \frac{d}{dt}(\delta\overline{z}_2)^2 &\leq -2\rho\delta\overline{z}_2^2\\ \frac{d}{dt}(\delta\overline{z}_3)^2 &\leq |\omega_2 + K_2|\delta\overline{z}_1^2 + (-2\omega_2 - 2K_2 + |\omega_2 + K_2|)\delta\overline{z}_3^2\end{aligned}$$

From the above inequalities, we obtain the quasi-monotone non-decreasing (off-diagonal entries non-negative) comparison system, we consider first the system (23) and obtain the comparison system, $\dot{w} = Gw$, *where*

$$G = \begin{bmatrix} (1 - 2K_1) & 1 & 0 \\ 0 & -2\rho & 0 \\ |\omega_2 + K_2| & 0 & (-2\omega_2 - 2K_2 + |\omega_2 + K_2|) \end{bmatrix}$$

We select the gains $K_1 = 5$ *and* $K_2 = 100$ *to make the above matrix* G *Hurwitz (contracting) and* $-KC \geq 0$. *Hence, the original dynamics (23) is contracting. Similarly, the dynamics (24) can be proved to be contracting. Thus, system (23) and (24) is an interval observer for the system (21). The simulation results are shown in Figure 3 with* $2 \leq z_1(0) \leq 5$, $1 \leq z_2(0) \leq 4$, $3 \leq z_3(0) \leq 8$, $z(0) = [4, 2, 6]^\top$, $d(t) = \sin(t)$, $\underline{d} = -1$ *and* $\overline{d} = 1$.

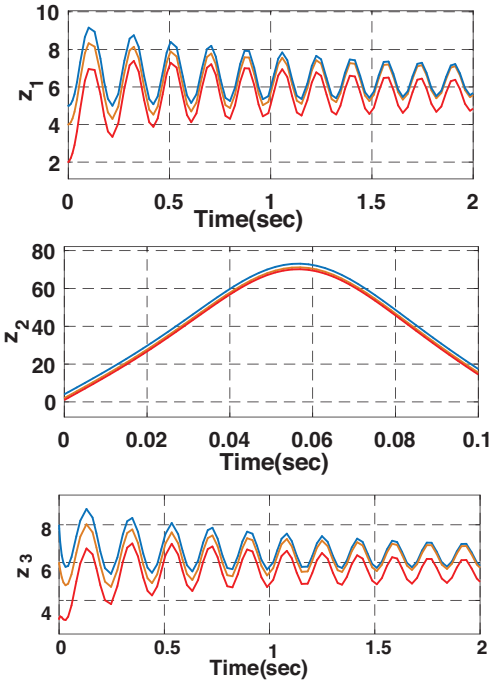

FIGURE 3 Interval observer for system (21)

Example 3. *Consider the modified self-excited nonlinear oscillator*

$$\begin{aligned}\dot{z}_1 &= z_2 + d(t)\\ \dot{z}_2 &= -\omega_1^2 \sin(z_1) - \rho z_2 + k_1 \arctan(k_2(z_1 - z_3)) + d(t)\\ \dot{z}_3 &= \omega_2 z_3 + d(t)\end{aligned} \tag{26}$$

with outputs $y_1 = z_1$ *and* $y_2 = z_1 - z_3$, *where* $z = [z_1, z_2, z_3]^\top \in \mathbb{R}^3$ *is the state vector,* $d(t)$ *is the unknown disturbance with* $\underline{d} \le d(t) \le \overline{d}$. *Parameter values:* $\omega_1 = \omega_2 = 40, k_1 = 5, k_2 = 10$ *and* $\rho = 1$.

We write the above system in the form

$$\dot{z} = Az + f(y) + Bd(t) \tag{27}$$

with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & \omega_2 \end{bmatrix},\ f(y) = \begin{bmatrix} 0 \\ -\omega_1^2 \sin(y_1) + k_1 \arctan(k_2 y_2) \\ 0 \end{bmatrix}$$

and $B = [1, 1, 1]^\top$. *The system matrix* A *is Metzler but not Hurwitz matrix. Now, we design interval observer as follows:*

$$\dot{\overline{z}} = A(y)\overline{z} + K(y - \overline{y}) + f(y) + B\overline{d} \tag{28}$$

$$\dot{\underline{z}} = A(y)\underline{z} + K(y - \underline{y}) + f(y) + B\underline{d} \tag{29}$$

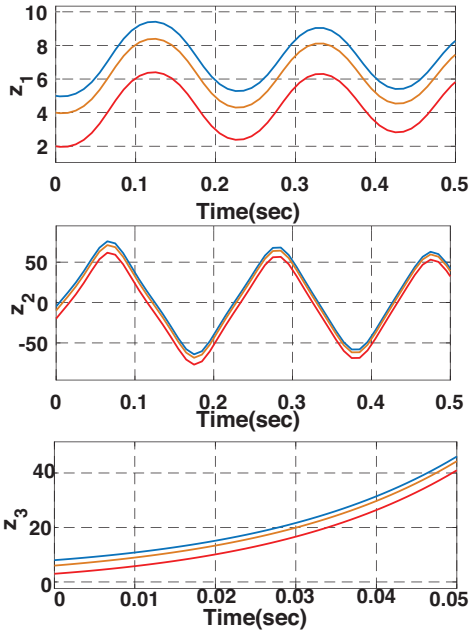

FIGURE 4 Interval observer for system (26)

where $K = \begin{bmatrix} K_1 & 0 \\ 0 & 0 \\ 0 & -K_2 \end{bmatrix}$ *is the gain matrix and* $y = Cz$, $C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$. *In a similar way, we consider first the system (28) and obtain the comparison system,* $\dot{w} = Gw$, *where*

$$G = \begin{bmatrix} (1 - 2K_1) & 1 & 0 \\ 0 & -2 & 0 \\ |K_2| & 0 & (2\omega_2 - 2K_2 + |K_2|) \end{bmatrix}$$

We select the gains $K_1 = 5$ *and* $K_2 = 100$ *to make the above matrix* G *Hurwitz (contracting) and* $-KC \ge 0$. *Hence, the original dynamics (28) is contracting. In a similar way, the dynamics (29) can be proved to be contracting. Thus, the system (28) and (29) is an interval observer for the system (26). The simulation results are shown in Figure 4 with* $2 \le z_1(0) \le 5$, $-20 \le z_2(0) \le -5$, $3 \le z_3(0) \le 8$, $z(0) = [4, -10, 6]^\top$, $d(t) = 0.5 \sin(t)$, $\underline{d} = -0.5$ *and* $\overline{d} = 0.5$.

Example 4. *Consider a model of an electromechanical system ([27])*

$$\begin{aligned}\dot{z}_1 &= z_2 + d(t)\\ \dot{z}_2 &= b_1 z_3 - a_1 \sin(z_1) - a_2 z_2 + d(t)\\ \dot{z}_3 &= b_0 u - a_3 z_2 - a_4 z_3 + d(t)\end{aligned} \tag{30}$$

with output $y = z_1$, *where* $z = [z_1, z_2, z_3]^\top \in \mathbb{R}^3$ *is the state vector,* $d(t)$ *is the unknown disturbance with* $\underline{d} \le d(t) \le \overline{d}$. *Parameters values are* $b_0 = 40, b_1 = 15, a_1 = 35, a_2 = 0.25, a_3 = 36$ *and* $a_4 = 200$.

We write the above system (30) as follows

$$\dot{z} = Az + f(y,u) + Bd(t) \tag{31}$$

$$with\ A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -0.25 & 15 \\ 0 & -36 & -200 \end{bmatrix},\ f(y,u) = \begin{bmatrix} 1 \\ -35\sin(y) \\ 40u \end{bmatrix}$$

and $B = [1,1,1]^\top$. *It is observed that the system matrix* A *is not a Metzler matrix, but a Hurwitz matrix.*

We design control u *to make the system (30) to be asymptotically stable using vector based contraction theory. To start with, consider the system (30) with* $d(t) = 0$, *let the input to the first subsystem be* $z_2 = -z_1$ *to obtain the subsystem* $\dot{z}_1 = -z_1$ *to be exponentially stable. Let the deviation variable be* $q_1 = z_2 + z_1$ *to transform the system into the structure*

$$\dot{z}_1 = q_1 - z_1$$

$$\dot{q}_1 = b_1 z_3 - a_1 \sin(z_1) + 0.75q_1 - 0.75z_1$$

Let us select the control $z_3 = \frac{1}{b_1}(a_1 \sin(z_1) - 2.75q_1 + 0.75z_1)$ *to obtain the subsystem* $\dot{q}_1 = -2q_1$ *to be exponentially stable. Let us again select the derivation variable as* $q_2 = z_3 - \frac{a_1}{b_1}\sin(z_1) + \frac{2.75q_1}{b_1} - \frac{0.75}{b_1}z_1$ *to transform the system into*

$$\begin{aligned} \dot{z}_1 &= q_1 - z_1 \\ \dot{q}_1 &= q_2 - 2q_1 \\ \dot{q}_2 &= b_0 u - \left(a_3 + \frac{6.25}{b_1}\right) z_2 - a_4 z_3 - \frac{a_1}{b_1}\cos(z_1) z_2 - \frac{5.5}{b_1} z_1 \\ &\quad + \frac{2.75}{b_1} q_2 \end{aligned} \tag{32}$$

The derivative of the vector valued distance (assuming L *as* $diag(\mathbb{1})$*) along the trajectories of the virtual dynamics of Equation (32) is given by*

$$\frac{d}{dt}(\delta z_1)^2 \le -\delta z_1^2 + \delta q_1^2$$

$$\frac{d}{dt}(\delta q_1)^2 \le -3\delta q_1^2 + \delta q_2^2$$

$$\begin{aligned} \frac{d}{dt}(\delta q_2)^2 = 2\delta q_2 \Bigg(& b_0 \delta u - \left(a_3 + \frac{6.25}{b_1}\right)\delta z_2 - a_4 \delta z_3 - \frac{5.5}{b_1}\delta z_1 \\ & - \frac{a_1}{b_1}(-\sin(z_1) z_2 \delta z_1 + \cos(z_1)\delta z_2) + \frac{2.75}{b_1}\delta q_2 \Bigg) \end{aligned}$$

We select

$$\begin{aligned} \delta u = \frac{1}{b_0} \Bigg(& \left(a_3 + \frac{6.25}{b_1}\right)\delta z_2 + a_4 \delta z_3 + \frac{5.5}{b_1}\delta z_1 \\ & + \frac{a_1}{b_1}(-\sin(z_1) z_2 \delta z_1 + \cos(z_1)\delta z_2) - \frac{2.75}{b_1}\delta q_2 - a_4 q_2 \Bigg) \end{aligned}$$

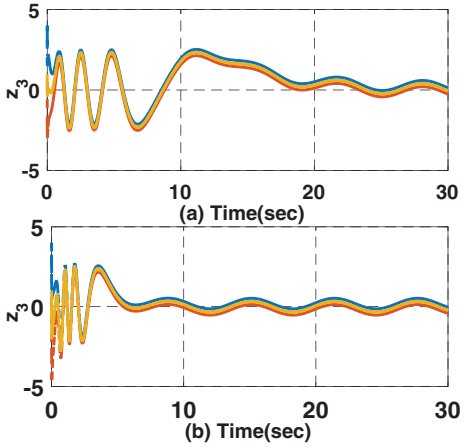

FIGURE 5 Interval observer for system (30) using (a) method in ref. [12] and (b) proposed method

to obtain the following linear comparison system

$$\dot{w}_1 = -w_1 + w_2$$

$$\dot{w}_2 = -3w_2 + w_3$$

$$\dot{w}_3 = -2a_4 w_3$$

to be quasi-monotone non-decreasing (off-diagonal entries non-negative) and contracting. Hence, the system (30) with $d(t) = 0$ *is contracting and thus the system trajectories converge to indeed an equilibrium point (origin) with the obtained control* $u(y,z)$ *(obtained by integrating* δu*),*

$$\begin{aligned} u(y,z) = \frac{1}{b_0}\Bigg(& \left(a_3 + \frac{(6.25)}{b_1} - 36.7\right) z_2 + \left(\frac{5.5}{b_1} - 26.69\right) z_1 + \\ & \frac{2a_1}{b_1}\cos(z_1) z_2 + \frac{a_1}{b_1}\left(a_4 + \frac{2.75}{b_1}\right)\sin(z_1) - \frac{2.75}{b_1} z_3 \Bigg). \end{aligned}$$

Now, the interval observer is formulated using time-invariant transformation as discussed in ref. [12] to transform the system matrix into a Metzler matrix. And, from Theorem 2, the formulated interval observer is asymptotically stable with the control $u(y,\bar{z})$. *We compare the performance of the interval observer with the control designed in ref. [12] and the proposed control* $(u(y,\bar{z}))$*, using state bounds from the interval observer itself and the system outputs considering* $d(t) = \frac{1}{9}\sin(t)$, $\underline{d} = -\frac{1}{9}$ *and* $\overline{d} = \frac{1}{9}$.

From the Figure 5, it can be noticed that the convergence time is less in the case of the proposed method as compared to the method in ref. [12]. Moreover, it does not require the Lyapunov candidate function formulation to show asymptotic stability of the interval observer.

5 | CONCLUSION

An interval observer has been designed for a class of nonlinear systems by the exploitation of the recently developed

theory known as vector based contraction theory. It provides an amenable feature that it does not require the formulation of Lyapunov candidate function to show stability, since there is no idea about its formulation, and does not require the construction of error dynamics to prove that the estimation error converges to zero. Dynamic output feedback control has been designed using state bounds from the constructed interval observer to prove it to be globally asymptotic stable. In the end, examples were illustrated to show the efficacy of the theoretical results. The proposed methodology can be extended to design interval observers for the systems having delays in the future.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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