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# APPLICATION OF THE POD METHOD TO NONLINEAR DYNAMIC ANALYSIS OF REINFORCED CONCRETE FRAME STRUCTURES SUBJECTED TO EARTHQUAKES

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## Abstract

*For the purpose of reducing computational cost, this paper presents an extension of the Proper Orthogonal Decomposition method (POD) to nonlinear dynamic analysis of reinforced concrete multistory frame structure. The material nonlinearity, due to plasticity and damaging, is modeled by the multifiber section approach. An initial expensive full model nonlinear time-history analysis is conducted for the frame structure subjected to a portion of an initial seismic base excitation. Then, POD modes are extracted and used to reduce the dynamic model for the rest of the initial base excitation or for multiple different ones. By comparing the full and reduced models, the computational cost was lowered by up to 94 % while maintaining good level of accuracy.*

**Keywords:** Reinforced concrete beam, Material nonlinearity, Multifiber section, Dynamic analysis, Reduced model, Proper Orthogonal Decomposition.

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## 1 INTRODUCTION

In structural seismic design, engineered structures should be checked for multiple accelerograms of previously recorded earthquakes in the region that are likely to reoccur and for all possible directions of vibration [1]. Performing dynamic time-history analysis through direct time integration techniques is computationally demanding and the presence of material nonlinearities in the structure further increases the already expensive computational cost. For this reason, several model reduction techniques were proposed in the literature to reduce the computational cost of the dynamic time-history analysis.

Modal truncation is classically applied for linear systems (or moderately nonlinear) systems, by considering only the most influential vibration modes of the structure in the truncated modal basis that is used to reduce the dynamic system [2-3]. For structures where nonlinear effects are significant, the traditional modal truncation approach proves to be

unreliable. Researchers inspired by the work of [4] have tried to make an analogy between nonlinear normal modes and linear ones. However, this nonlinear modal analysis was not very successful due to its limitations when non-smooth nonlinearities are present in the structure [5].

For seismic analysis considering nonlinear material behavior of reinforced concrete structures, the pushover analysis is commonly applied. This static nonlinear approach solely considers the fundamental vibration mode of the structure in the direction under consideration [6]. Horizontal loads representing earthquake effects are distributed on the structure proportionally to its fundamental mode shape vector and are increased progressively while conducting a nonlinear calculation. This approach provides only maximum response values and is only applicable to structures where the fundamental mode shape is the dominant mode of vibration and is thus limited to regular low-rise buildings. Alternative pushover analyses have been proposed to improve the estimation of the structural maximum response [7-8] but do not provide time-dependent responses.

In the early 20<sup>th</sup> century, a statistical approach, known under many names “Proper Orthogonal Decomposition” (POD), “Principal Component Analysis” (PCA) or “Karhunen-Loève Decomposition” (KLD) is proposed to reduce the computational cost of dynamic analysis [9]. Nowadays, this method is used in a wide variety of fields where large numerical models need to be reduced such as turbulent flow in fluid mechanics, dynamic models for microelectromechanical systems, structural dynamic models, etc... Though the POD method is linear by nature, it is frequently applied to reduced order modeling of nonlinear structural systems [10]. However, there are few examples of POD method being applied to nonlinear problems in earthquake engineering. In [11], Kerschen and Golinval applied this model order reduction method to a 3D portal frame with nonlinear stiffness elements. Bamer and Bucher applied it to a 2D frame system with friction beam elements in [12] and a 3D wall-frame structure with friction pendulum elements in [13]. To evaluate properly seismic demand on structures, there is currently a need for more accurate models, which explicitly account for the geometrical and material nonlinearities naturally present in reinforced concrete (RC) structures. To the authors’ best knowledge, the POD method was never used for the dynamic analysis of RC structures with material nonlinearities. The goal of this paper is to present a reduction strategy based on the POD method to study reinforced concrete multistory frame structures subjected to seismic base excitations. In this work, material nonlinearity is modeled by the multifiber section approach. This paper demonstrates the efficiency of the proposed POD Reduced Order Models (POD-ROM) approach in cutting down the computational cost for the nonlinear structural time-history analysis.

Section 2 goes through the commonly used nonlinear structural models and justifies the choice of the multifiber section approach. Section 3 focuses on the full and reduced dynamic analysis of structures in addition to the POD techniques used for lowering the computational cost of a RC multistory frame structure with material nonlinearities. Section 4 presents numerical applications of the POD based time reduction techniques while assessing their accuracy and efficiency in time saving.

## **2 NONLINEAR DISCRETIZED MODEL**

### **2.1 Modeling approach**

During earthquakes, RC structures generally undergo nonlinear deformations. On one hand, an advanced nonlinear modeling is required to predict accurately the structural behavior. On the other hand, a detailed modeling of nonlinear RC structures is very computationally expensive. Several approaches are available in the literature to model

nonlinear RC structures, with element models and constitutive models of various complexity [14-15]. A good compromise between accuracy and time efficiency is to consider frame structures with Euler-Bernoulli multifiber beam elements. This is the approach considered in this work, allowing distributed plasticity with an acceptable computational effort.

This multifiber approach consists in dividing the structural element cross section into a set of longitudinal fibers. For reinforced concrete structures, the fibers can be made of steel reinforcements, confined or unconfined concrete (refer to Figure 1). Each fiber has the potential to undergo nonlinear inelastic longitudinal deformation according to the uniaxial stress-strain behavior of its corresponding material [16-22]. As a result, material nonlinearity is distributed all over the structural element length and cross section.

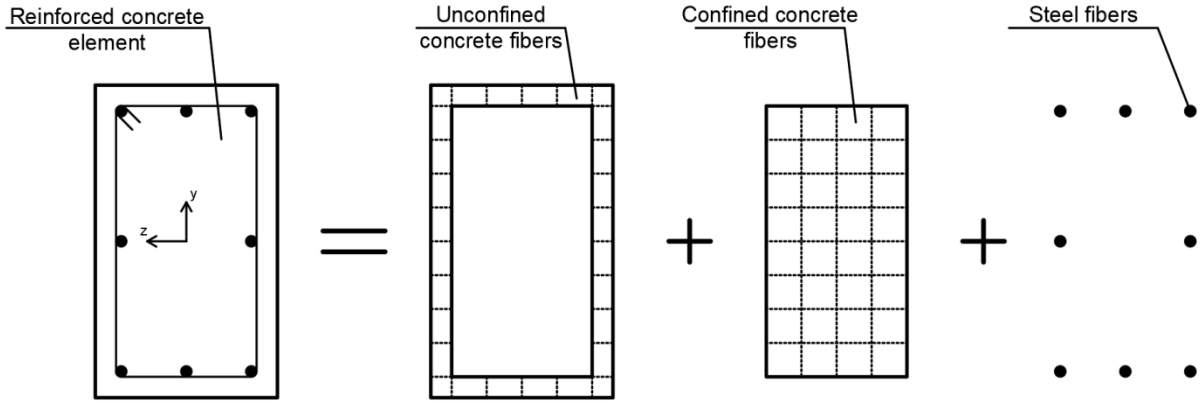


Figure 1. Multifiber reinforced concrete section

## 2.2 Finite element discretization

After finite element discretization, the dynamic equation of the nonlinear system is:

$$[M]\{\ddot{X}(t)\} + [C]\{\dot{X}(t)\} + \{F_{NL}(\{X(t)\})\} = \{F(t)\} \quad (1)$$

where  $\{X(t)\}$ ,  $\{\dot{X}(t)\}$  and  $\{\ddot{X}(t)\}$  are respectively the displacement, velocity and acceleration nodal vectors.  $[M]$  is the mass matrix,  $[C]$  is the damping matrix, and  $\{F(t)\}$  is the excitation force vector applied on the system.  $\{F_{NL}\}$  is the nonlinear nodal force vector defined by the following integral over the finite element volume  $V$ :

$$\{F_{NL}\} = \int_V [B(x)]^T \begin{Bmatrix} 1 \\ -y \end{Bmatrix} \sigma_{fiber}(x, y) dV \quad (2)$$

where  $(x, y, z)$  are local coordinates in the reference system as indicated in Figure 1,  $[B(x)]$  is the gradient operator containing the derivatives of the linear and cubic shape functions and  $\sigma_{fiber}$  is the axial stress of a given fiber described by the corresponding constitutive equation.

The dynamic structural response is generally computed from equation (1) by direct-integration time-history analysis. In this approach, temporal discretization is considered and the direct time-integration is conducted using implicit methods like Newmark- $\beta$  [23], Wilson  $\theta$  [24], HHT- $\alpha$  [25] or explicit methods like central difference and Runge-Kutta. The Newmark- $\beta$  method has the advantage of being unconditionally stable when the constant average acceleration is assumed. Therefore, it will be considered in this work along with the Newton-Raphson method for the full-order model nonlinear calculation.

### 3 REDUCED-ORDER MODEL (ROM)

In reduced-order models, the dynamic structural response of a system with  $m$  degrees of freedom  $\{X(t)\}$ , is reduced to  $s$  degrees:

$$\{X(t)\} \cong [T]\{Q(t)\} \quad (3)$$

where  $[T] \in \mathbb{R}^{m \times s}$  ( $s \ll m$ ) is a transformation matrix and  $\{Q(t)\} \in \mathbb{R}^s$  is the reduced coordinate vector.

Replacing  $\{X(t)\}$  by  $[T]\{Q(t)\}$  in equation (1) and multiplying both sides by  $[T]^T$  lead to:

$$[T]^T[M][T]\{\ddot{Q}(t)\} + [T]^T[C][T]\{\dot{Q}(t)\} + [T]^T\{F_{NL}([T]\{Q(t)\})\} = [T]^T\{F(t)\} \quad (4)$$

The initially  $m$  degrees of freedom dynamic system is thus reduced to  $s$  degrees of freedom. However, the nonlinear restoring force  $\{F_{NL}([T]\{Q(t)\})\}$  cannot be reduced and always needs to be calculated in the full coordinate model which makes this step the most time-consuming part of the entire process. Due to this setback and in order to maintain an effective time saving, the need for calculating the nonlinear restoring force should be kept minimum during the direct time-integration analysis of the reduced model.

Implicit direct time-integration techniques are usually used in conjunction with the Newton-Raphson approach for solving nonlinear systems. At each time step, multiple iterations are needed to achieve convergence. For every iteration, the calculation of the tangent stiffness matrix and the nonlinear restoring force is required which is time-consuming. Using the modified or constant stiffness Newton-Raphson approaches will respectively reduce or stop the need for the tangent stiffness calculation. However, the number of iterations required for convergence at each time step will grow thus increasing the number of calls for nonlinear restoring force calculation.

On the other hand, for explicit direct time-integration techniques, the popular central difference method requires no iterations per time step and no expensive calculation of the tangent stiffness matrix. Only the expensive nonlinear restoring force is calculated once per time step. However, the central difference approach is conditionally stable and needs to satisfy the following stability condition [26]:

$$\Delta t < \frac{2}{\omega_{max}} \quad (5)$$

where  $\Delta t$  is the time step and  $\omega_{max}$  is the largest natural pulsation of the system. In full model analysis, the structure has a relatively large number of degrees of freedom which will result in high natural pulsations (for high vibration modes). In order to satisfy the stability condition, relatively small time steps should be adopted for the explicit central difference method and thus increasing the computational cost. Therefore, the implicit Newmark- $\beta$  method will be considered for full dynamic model analysis in this work. On the other hand, for reduced model analysis, the reduced structure has significantly fewer number of degrees of freedom which will yield relatively smaller natural pulsations. Thus, it is possible to use larger time steps in the explicit central difference method while respecting the stability condition. As a consequence, this explicit central difference method is very effective and will be used in this work for reduced-order models. More details on direct time integration techniques for nonlinear dynamic structural systems can be found in [26].

The transformation matrix used to reduce the dynamic structural system is constructed such that the error  $\varepsilon$  is small:

$$\varepsilon(t) = \|\{X(t)\} - [T]\{Q(t)\}\| \quad (6)$$

In this work, two reduced-order models are studied (and compared):

- The Modal Reduced-Order Model (M-ROM), based on the modal truncation method.
- The POD Reduced-Order Model (POD-ROM), based on the Proper Orthogonal Decomposition method.

### 3.1 Modal Reduced-order model (M-ROM)

A classical linear modal analysis is conducted on the structure. Then the reduction base  $[T]$  is defined based on the conventional modal truncation criteria (first  $s$  linear modes with cumulated effective modal mass exceeding 90% of the total structural mass). The dynamic nonlinear structural model is reduced by projection on the base  $[T]$ . It should be noted that in this dynamic analysis and due to the nonlinear return force  $F_{NL}(\{X(t)\})$ , the linear modes can no longer be decoupled.

### 3.2 POD Reduced-order model (POD-ROM)

The Proper Orthogonal Decomposition (POD) is a data-driven method based on the statistical Principal Component Analysis (PCA) of observation dataset. Let's consider a data matrix  $[X]$  containing  $n$  observation vectors  $[X] = [\{X_1\} \ \cdots \ \{X_n\}]$  and each observation vector is made of  $m$  dimensions

$$[X] = [\{X_1\} \ \cdots \ \{X_n\}] = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \quad (7)$$

It can be demonstrated that the principal component vectors of the data set  $[X]$  are the eigenvectors of  $[X][X]^T$  called POD modes and their corresponding eigenvalues  $\lambda$  are called Proper Orthogonal Values (POV). These Eigenvalues represent the quantity of action occurring on the corresponding eigenvectors. The higher the eigenvalue is the more action we have on the corresponding eigenvector and the more energy it holds. Therefore, the higher the POV is, the more essential its corresponding POD mode is in recreating  $[X]$ . More information on the principal component analysis can be found in [27].

The reduction base  $[T]$  is generally defined by POD modes truncation, the first  $s$  POD modes carrying at least 99% of the total system energy (the POV represents the corresponding POD mode energy) are considered

$$\frac{\sum_{i=1}^s \lambda_i}{\sum_{j=1}^m \lambda_j} \geq 99\% \quad (8)$$

However, this criterion is not sufficient. In fact, for a better POD modes truncation, the cumulated modes' energy should be considered and each supplementary mode have to be assessed for its additional energy regardless of the cumulated energy percentage. In other words, truncation is made where the cumulated POV curve in function of the mode number  $f(s) = \sum_{i=1}^s \lambda_i$  forms an elbow transition from an increasing part to a relatively stable part (refer to Figure 2).

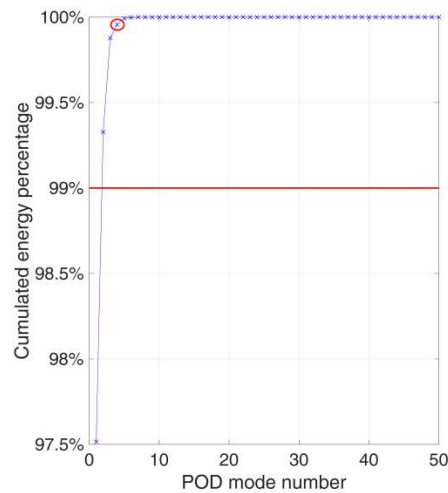


Figure 2. Example of POD modes truncation

In structural dynamics, the snapshot matrix (data matrix containing observations of system displacement) requires the displacement vectors at selected time intervals of the full order dynamic model. Since there is no escape from this initial time consuming full model analysis, various techniques have been proposed to benefit from the POD modes after conducting the initial costly analysis. In [12], the dynamic analysis cost of a steel frame structure with nonlinear seismic base isolation is reduced by initially conducting the expensive full model analysis on a portion of the base excitation. Then, snapshots were taken, the POD modes calculated, the dynamic system reduced and the response corresponding to the remaining part of the base excitation was calculated using the reduced-order model. As previously mentioned, for a structural seismic design, the structure is studied for a range of possible earthquakes and is analyzed and checked for each excitation (earthquake record) separately. So, [13] reduced the analysis cost of a reinforced concrete structure with nonlinear seismic base isolation subjected to multiple base excitations. First, they conducted the expensive full model analysis for only one base excitation. Then they extracted the snapshot matrix, calculated the POD modes and used the reduced-order dynamic model for the calculation of the remaining base excitations. Neither [12] nor [13] have tried reducing the dynamic model of a reinforced concrete structure with material nonlinearities. In this article, the before mentioned time saving techniques will be extended and used on reinforced concrete multistory frame structure where the material nonlinearity is modeled by the multifiber section approach.

#### 4 APPLICATION

In this section, the transient dynamic response of the frame structure subjected to seismic base excitations is studied. The objective is to demonstrate the efficiency of the POD Reduced Order Model (POD-ROM) in reducing the numerical cost.

The following 4 earthquake recordings obtained from the Center of Engineering for Strong Motion Data CESMD [28] were considered (refer to Table 1 and Figure 3).

Earthquake	Location	Date	Magnitude	Measurement	Vibration	Total	Time
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				station	direction	duration	step
Northridge	Los Angeles, USA	01/17/1994	6.4 ML	Newhall LA county fire station	0°	60s	20ms
Elcentro	California, USA	05/18/1940	6.9 Mw	Elcentro	0°	53.74s	20ms
L'Aquila	L'Aquila, Italy	04/06/2009	6.3 Mw	L'Aquila V.Aterno Centro Valle	90°	60s	20ms
Chile	Off the coast of central Chile	02/27/2010	8.8 Mw	Constitucion city	90°	120s	20ms

Table 1: Considered earthquakes.

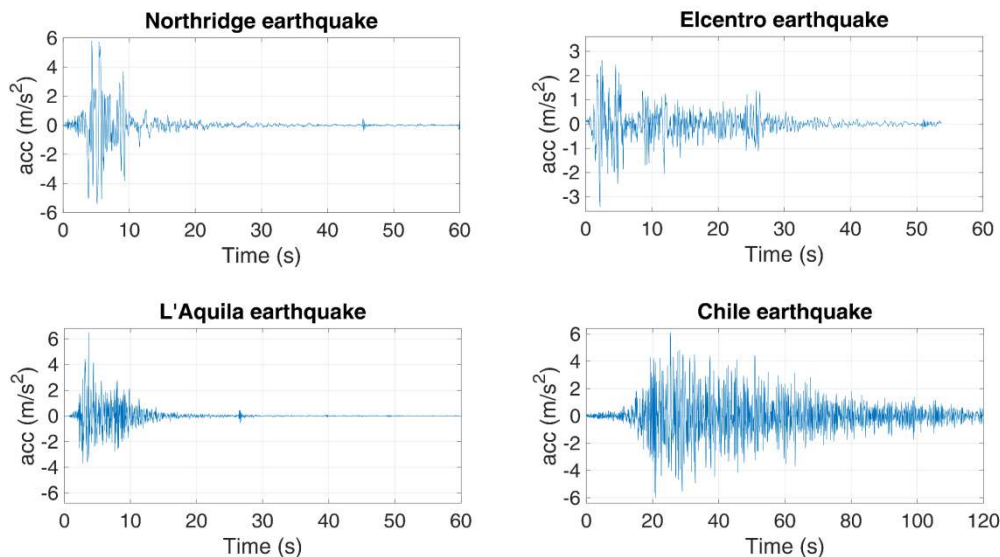


Figure 3. Earthquakes accelerograms

#### 4.1 Description of the investigated benchmark structure

The considered structure is a 2D Reinforced Concrete (RC) multistory frame made up of 10 stories and 5 spans with a 3 m story height, a 5 m span length, a 4 t/m uniform load is applied on beams and the structural self-weight is neglected (refer to Figure 4). All concrete columns and beams are discretized into 1 m long finite elements of identical cross section 40x40 cm, with four 20 mm High Bond HB reinforcing bars at both top and bottom sides (refer to Figure 5). The cross section is composed of 4 concrete fibers and 2 steel fibers. Rayleigh damping was used to set a 5% damping ratio for the first two eigenmodes (satisfying the classic linear modes truncation criterion, more than 90% of the total mass is participating in the first two eigenmodes).

Nonlinear material deformations occur in the most stressed zones. For the case of a frame structure subjected to seismic loads, these zones are the columns and beams extremities in lower floors. Thus, nonlinear elements of the benchmark structure are considered near the



beam column connections at the first 5 stories (refer to Figure 6). The bilinear backbone curve is adopted to model the reinforcing steel bars. Under cyclic loading, if nonlinearity is reached, the steel material will undergo a kinematic hysteresis behavior (for further details refer to Table 2 and Figure 7).

Material	Elastic Young Modulus	Yield stress	Yield strain	Ultimate stress	Ultimate strain
Rebar	200 GPa	400 MPa	2‰	420 MPa	2.5%

Table 2: Steel rebar characteristics

Concrete is unconfined and modeled according to a simplified version of Mander model [29] that takes into account the damaging phenomena (for further details refer to Table 3 and Figure 8).

Material	Elastic Young Modulus	Maximum compressive stress	Strain at maximum compressive stress	Ultimate compressive stress	Ultimate compressive strain	Ultimate tensile stress	Ultimate tensile strain
Concrete	25 GPa	25 MPa	2‰	20 MPa	4‰	2.5 MPa	0.1‰

Table 3: Concrete characteristics

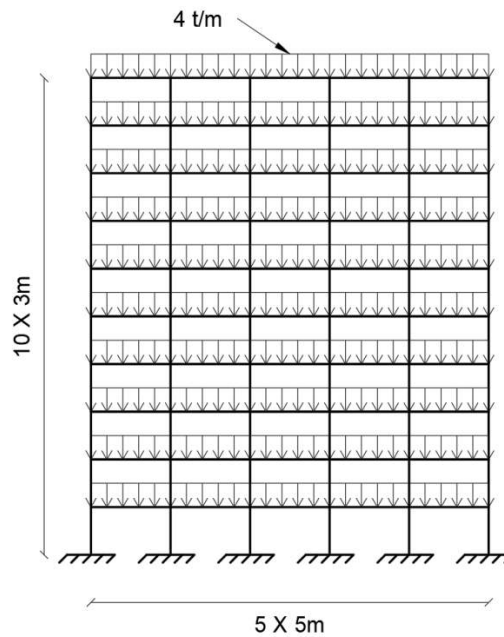


Figure 4. RC frame geometry and loading

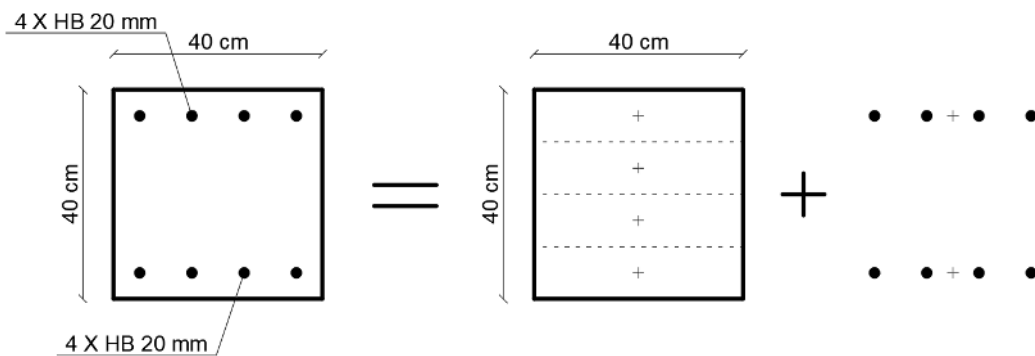


Figure 5. RC element multifiber section

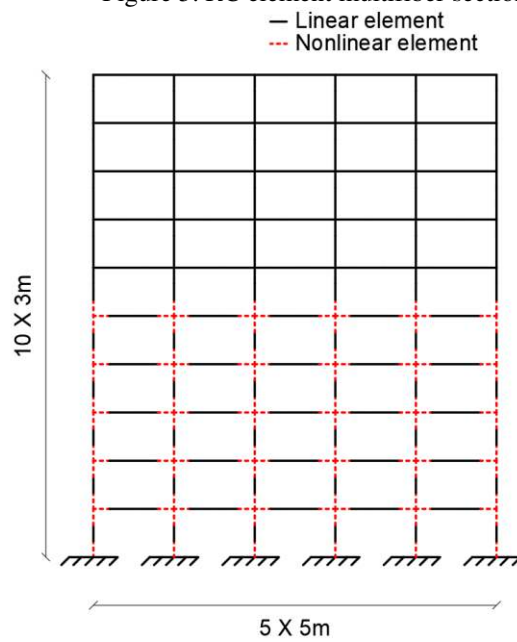


Figure 6. Position of nonlinear elements in RC frame

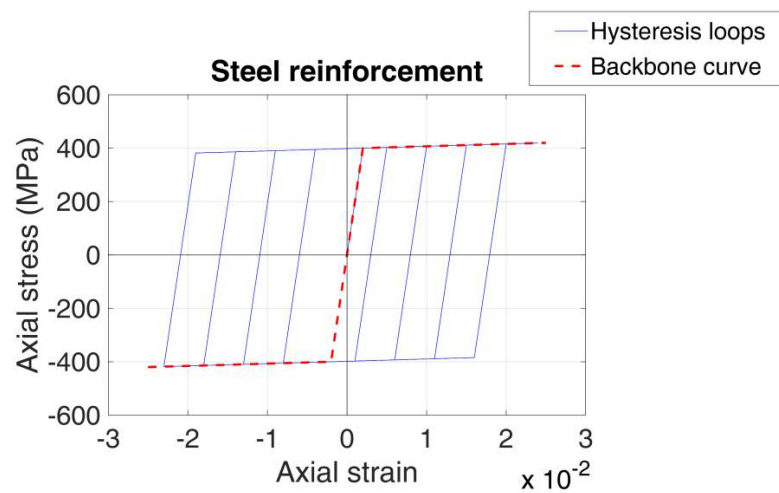


Figure 7. Steel reinforcement axial stress-strain curve

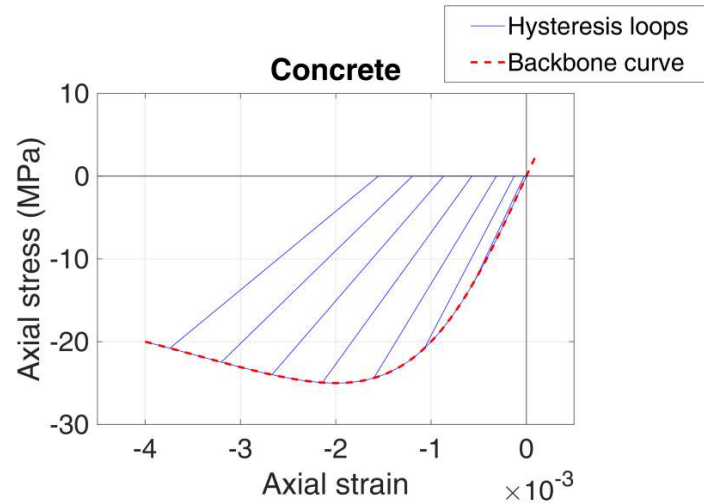


Figure 8. Concrete axial stress-strain curve

For linear dynamic properties of the structure, the first 5 vibration modes in the horizontal direction are presented in Table 4.

Mode	Period	Effective mass	Percentage of total mass
1	1.74 s	815.7 t	81.57%
2	0.57 s	97.4 t	9.74%
3	0.33 s	35.9 t	3.59%
4	0.23 s	19.3 t	1.93%
5	0.17 s	12.1 t	1.21%

Table 4: Linear dynamic properties of the RC frame

## 4.2 Results

Before investigating the efficiency of the POD in reducing nonlinear dynamic systems, it should be demonstrated that linear vibration modes are not effective for nonlinear problems. For this reason, a full model implicit nonlinear time-history analysis with a 20 ms time step was conducted for the Northridge earthquake. Then, 50 resulting displacement vectors were taken at equally spaced time intervals during the first 15 seconds of the vibration (where most of the powerful excitation occurs) and gathered in a snapshot matrix. POD modes were extracted from the snapshot matrix and the first 4 POD modes satisfied the truncation criterion. For the linear modes of vibration, the first two modes satisfy the classical truncation criterion of 90% effective modal mass participation. However, to make a fair comparison, the first 4 linear modes of vibrations were considered similarly to the POD modes (refer to Figure 9). This structure with initially 1140 degrees of freedom is reduced to only 4. Then, explicit nonlinear time-history analyses with a 20 ms time step were conducted using two reduced-order models respectively based on POD modes and linear eigenmodes (refer to Figure 10).

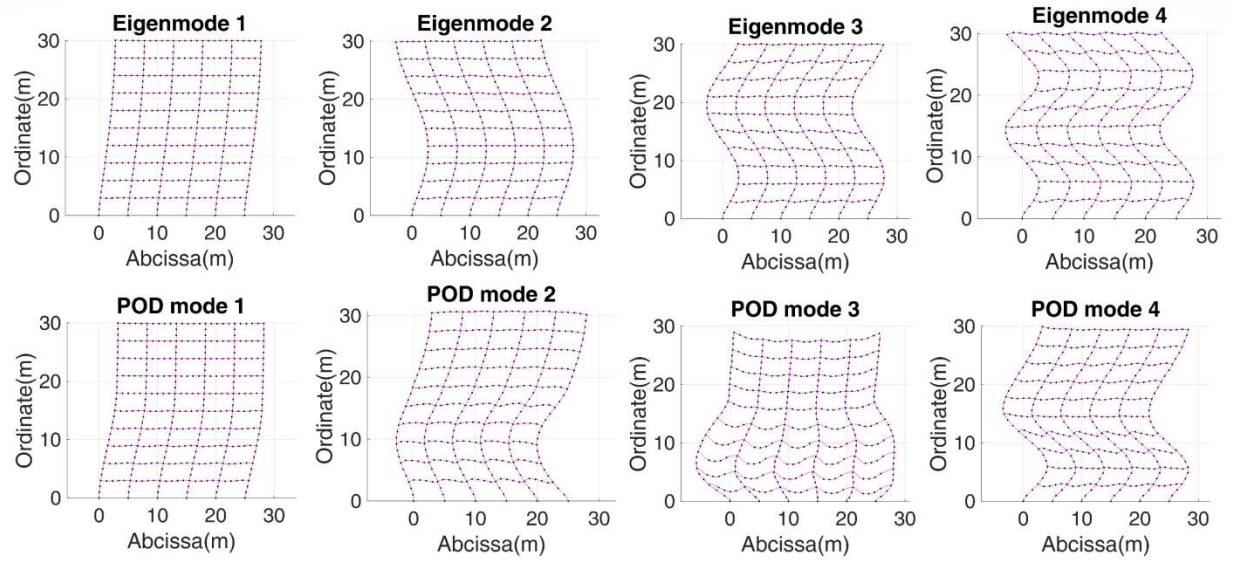


Figure 9. Classic structural eigenmodes Vs POD modes

By comparing, in Figure 9, the first 4 POD modes with the first 4 classical linear eigenmodes of the structure, we can clearly see the nonlinear behavior of the first 5 stories of the structure in the POD modes, especially for modes 2 and 3.

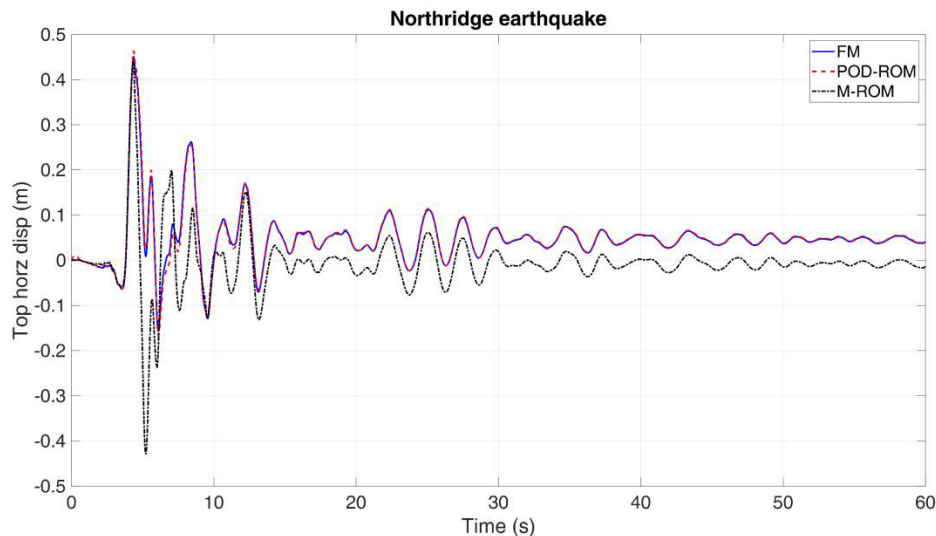


Figure 10. Structural top left corner horizontal displacement in function of time for Full Model (FM), POD Reduced-Order Model (POD-ROM) and Modal Reduced-Order Model (M-ROM) analyses

By studying the top left corner horizontal displacement of the structure subjected to the Northridge earthquake in function of time, it is clear in Figure 10 that the POD Reduced-Order Model (POD-ROM) is a lot more accurate than the Modal Reduced-Order Model (M-ROM). In fact, the average error for the POD-ROM is 0.28 cm while it is 5.98 cm for the M-ROM. Additional testing showed that the M-ROM required more than 500 linear modes to achieve the same accuracy obtained with only 4 POD modes.

Another remarkable aspect in Figure 10 is the shift between POD-ROM and M-ROM results starting from the 10<sup>th</sup> second. According to Figure 3, the vibration intensity of Northridge earthquake decreases at this instant. So, the structure returns to vibrating in the

linear domain with residual deformations and some damaged properties. At this low vibration intensity stage, projecting on initial linear elastic vibration modes produces less errors. However, this projection was not doing well in the previous high intensity vibration phase, so M-ROM analysis entered the low intensity vibration stage with a result offset that continued with it.

After demonstrating the accuracy of the POD modes, the efficiency of the POD-ROM in time reduction will be studied in the following. In a first approach (similar to [12]), each of the four earthquakes was studied separately. A full model implicit nonlinear time-history analysis with a 20 ms time step was conducted solely on the first quarter of the vibration time length. Then, 50 snapshots at equally spaced time intervals were taken from the resulting displacement data. Next, POD modes were extracted while respecting the truncation criterion and used for a reduced-order model explicit nonlinear time-history analyses with a 20 ms time step carried on the remaining three quarters of the vibration time length (refer to Figure 11).

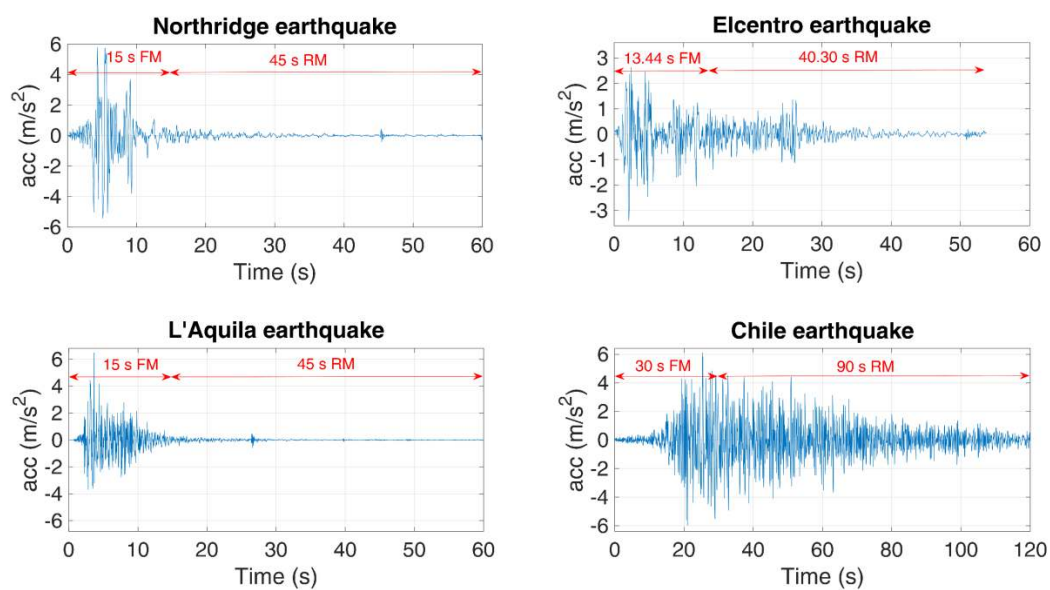


Figure 11. Time length distribution in accelerograms between Full Model (FM) and Reduced Model (RM) analyses

It should be noted that for comparison purpose, Full Model (FM) implicit nonlinear time-history analysis was conducted for each earthquake over its entire time length in order to have a base reference. In addition, the process of starting with FM analysis for the first quarter and then continuing with a Reduced Model (RM) analysis will be designated as Partially Reduced Model (PRM) analysis.

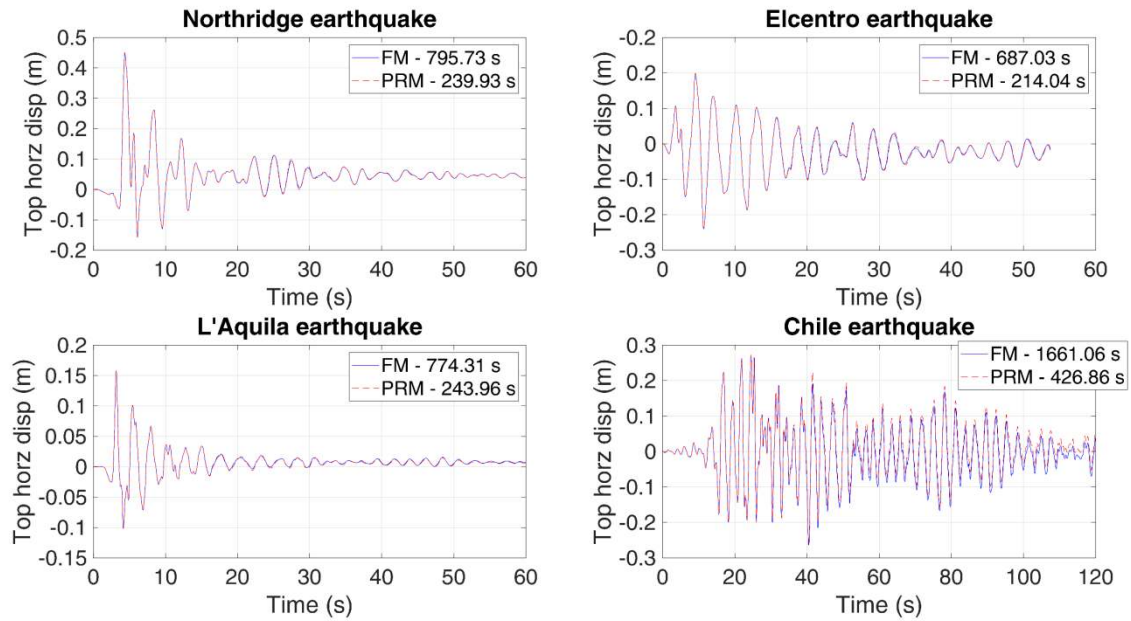


Figure 12. Structural top left corner horizontal displacement in function of time for Full Model (FM) and Partially Reduced Model (PRM) analyses

As shown in Figure 12, the PRM analyses results are very close to the FM results and at a reduced computational cost, for further details refer to Table 5.

Earthquake	FM time	PRM time	Time saving	Speedup	Average absolute displacement error	Max horiz displacement
Northridge	795.73 s	239.93 s	69.85%	3.3	0.12 cm	45.06 cm
Elcentro	687.03 s	214.04 s	68.85%	3.2	0.31 cm	23.96 cm
L'Aquila	774.31 s	243.96 s	68.49%	3.2	0.10 cm	15.85 cm
Chile	1661.06 s	426.86 s	74.30%	3.9	1.67 cm	27.24 cm

Table 5: Accuracy and time saving of the Partially Reduced Model (PRM) with respect to the Full Model (FM)

It is clear that projecting the system on POD modes reduces the computational cost of a nonlinear dynamic analysis while maintaining a good level of accuracy. However, the current application process always requires conducting the expensive FM analysis over a portion of the earthquake's time length and thus reducing the time saving efficiency of the approach. For a nonlinear dynamic analysis covering only a single accelerogram, nothing more can be done. On the other hand, for analyses covering multiple earthquake scenarios a second approach (similar to [13]) is possible.

For this second approach, Northridge earthquake is taken as the initial vibration since it contains strong vibrations at the beginning of its accelerogram (refer to Figure 3). Full Model (FM) with a 20 ms time step was carried out for the first 15 seconds of this vibration. Similarly to the first approach, 50 snapshots were taken, POD modes were extracted and the dynamic system was reduced (first 4 POD modes required to satisfy the truncation criterion). Then, Reduced Model (RM) explicit nonlinear time-history analyses with a 20 ms time step were carried out for the entire time length of all earthquakes (Northridge, Elcentro, L'Aquila and Chile).

Also here and for comparison purpose, Full Model analysis for each earthquake over its entire time length was considered for base reference.

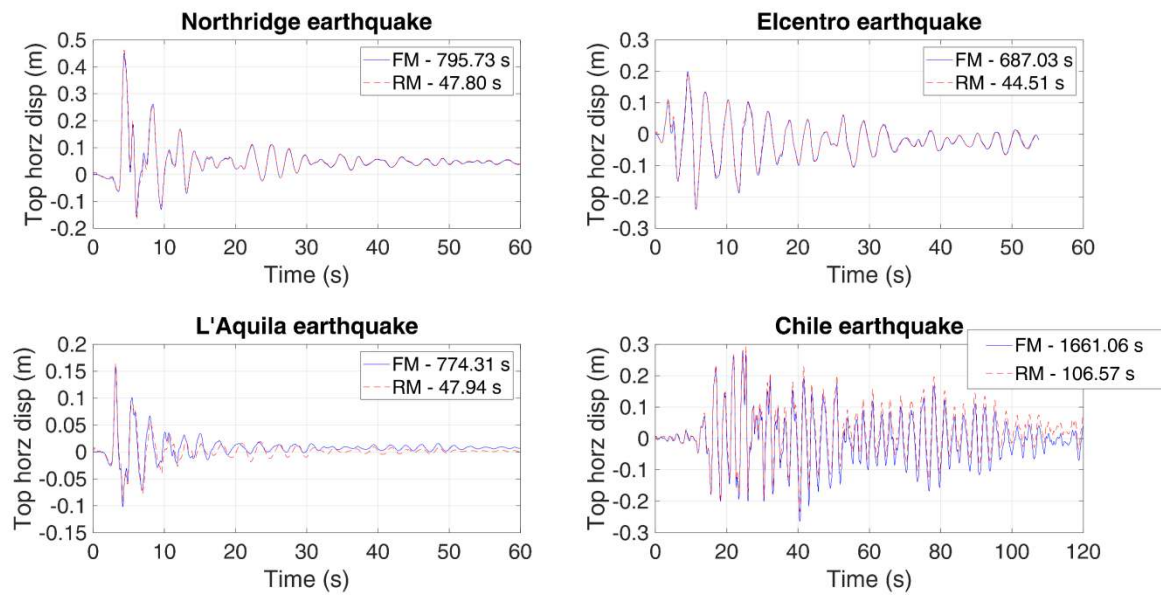


Figure 13. Structural top left corner horizontal displacement in function of time for Full Models (FM) and Reduced Models (RM) analyses

Figure 13 shows that the reduced model results are very close to the full models and at a fraction of the computational cost, for further details refer to Table 6.

Earthquake	FM time	RM time	Time saving	Speed up	Average absolute displacement error	Absolute error at max horiz displacement	Max horiz displacement
Northridge	795.73 s	47.80 s	93.99%	16.6	0.28 cm	1.26 cm	45.06 cm
Elcentro	687.03 s	44.51 s	93.52%	15.4	0.36 cm	0.24 cm	23.96 cm
L'Aquila	774.31 s	47.94 s	93.81%	16.2	0.79 cm	0.52 cm	15.85 cm
Chile	1661.06 s	106.57 s	93.58%	15.6	2.71 cm	0.85 cm	27.24 cm

Table 6: Accuracy and time saving of the Reduced Model (RM) with respect to the Full Model (FM)

The projection on POD modes has significantly reduced the computational cost of the nonlinear dynamic analysis. In addition, the POD modes extracted from the Full Model (FM) analysis of Northridge earthquake are working well in the reduction of the structural model subjected to other excitations (Elcentro, L'Aquila and Chile earthquakes). It should be noted that a reduced model calculation based on the implicit Newmark- $\beta$  method was also tested. It gave similar precision but with only 25% time saving. Thus, highlighting the importance of adopting central difference method for reduced models.

This second approach is a lot better in time reduction than the previous single earthquake one due to the fact that no computational costly FM analysis was required for Elcentro, L'Aquila and Chile earthquakes. However, the POD modes projection technique is criticized for being load dependent. The choice of the base excitation to consider for the extraction of the POD modes is suspected to have an influence on the accuracy of the reduction process.

Nevertheless, in structural seismic analysis, the dynamic load resulting from a base excitation always has the same pattern (proportional to the product of the mass matrix by the influence vector) regardless of the considered vibration. In addition and as demonstrated in this last application, choosing a base vibration that generates nonlinear behavior while taking

good representative snapshots will result in a POD base that can be applied for reducing the dynamic model when subjected to other excitations.

In Figures 12 and 13 and Tables 5 and 6, it is remarkable that reduced models were less accurate for Chile earthquake. This might be due to the fact that Chile earthquake data were measured close to the quake epicenter and in this case near fault effects are affecting the results. More information on this topic can be found in [30].

The model sensitivity to fiber discretization was investigated, tests were conducted with respectively 8 and 16 concrete fibers. No important effect was noticed on the precision and efficiency of reduction techniques.

Lastly, to overcome issues related to the choice of base excitation for POD modes extraction and the dependency of these modes on the applied loading, a synthetic base excitation constructed from the frequency spectrums envelop of considered earthquakes might be a good candidate for the FM analysis used to obtain the POD modes.

## 5 CONCLUSIONS AND PERSPECTIVES

In this paper, the application of the Proper Orthogonal Decomposition was extended to reduce the computational cost of nonlinear dynamic analysis. It was applied to a reinforced concrete multistory frame structure where the material nonlinearity was modeled by the multifiber section approach. A system containing more than 1000 degrees of freedom was successfully reduced to only 4 while maintaining an acceptable accuracy level and achieving a speedup of 16 for analysis covering multiple earthquake scenarios. It was also shown that POD modes obtained from the analysis of a full structural model subjected to a base vibration were also convenient for reducing the same model when subjected to different earthquakes. The key points here are (i) choosing an earthquake that generates a nonlinear response of the structure and (ii) having a snapshot matrix representing well the dynamics of the system.

A perspective for future work is to minimize the length of the seismic signal studied initially with full model analysis. This can be achieved by focusing the analysis around acceleration peaks of the signal. Also, a synthetic base excitation inspired from real earthquakes can be used for the calculation of the POD modes to decrease their dependency on the type of loading on the structure. Finally, the approach adopted in this work to deal with Reinforced Concrete (RC) beams and columns can be extended to RC plates, shells and membranes by using the layered 2D finite element model.

## REFERENCES

- [1] EC8-1, Eurocode 8: Design of structures for earthquake resistance - Part 1, European Committee for Standardization, 2004.
- [2] W. Larbi, J.-F. Deü, Reduced order finite element formulations for vibration reduction using piezoelectric shunt damping, *Applied Acoustics*, **147**, 111-120, 2019.
- [3] W. Larbi, J.-F. Deü, R. Ohayon, Finite element reduced order model for noise and vibration reduction of double sandwich panels using shunted piezoelectric patches, *Applied Acoustics*, **108**, 40-49, 2016.
- [4] R. Rosenberg, On Nonlinear Vibrations of Systems with Many Degrees of Freedom, *Advances in Applied Mechanics*, **9**, 155-242, 1966.
- [5] S. Shaw, P. Holmes, A periodically forced piecewise linear oscillator, *Journal of Sound and Vibration*, **90**, 129-155, 1983.



- 
- [6] H. Krawinkler, G. D. P. K. Seneviratna, Pros and cons of a pushover analysis of seismic performance evaluation, *Engineering structures*, **20**, 452-464, 1998.
- [7] S.P. Kim, Y. C. Kurama, An alternative pushover analysis procedure to estimate seismic displacement demands, *Engineering structures*, **30(12)**, 3793-3807, 2008.
- [8] R. Hasan, L. Xu, D. E. Grierson, Push-over analysis for performance-based seismic design *Computers & structures*, **80**, 2483-2493, 2002.
- [9] H. Hotelling, Analysis of a complex of statistical variables into principal components, *Journal of educational psychology*, **24(6)**, 417-441, 1933.
- [10] G. Kerschen, J. C. Golinval, A. F. Vakakis, L. A. Bergman, The method of proper orthogonal decomposition for dynamical characterization and order reduction of mechanical systems: an overview, *Nonlinear dynamics* **41**, 147-169, 2005.
- [11] G. Kerschen, J. C. Golinval, A model updating strategy of non-linear vibrating structures, *International Journal for Numerical Methods in Engineering*, **60(13)**, 2147-2164, 2004.
- [12] F. Bamer, C. Bucher, Application of the proper orthogonal decomposition for linear and nonlinear structures under transient excitations, *Acta Mechanica*, **223**, 2549-2563, 2012.
- [13] F. Bamer, A.K. Amiri, C. Bucher, A new model order reduction strategy adapted to nonlinear problems in earthquake engineering, *Earthquake Engineering and Structural Dynamics*, **46**, 537-559, 2017.
- [14] S. K. Kunnath, Modeling of reinforced concrete structures for nonlinear seismic simulation, *Journal of Structural Integrity and Maintenance*, **3(3)**, 137-149, 2018.
- [15] F. Mazza, M. Mazza, Nonlinear modeling and analysis of R.C. framed buildings located in a near-fault area, *Open Construction Building Technology Journal*, **6(1)**, 346-354, 2012.
- [16] K.M.S. Mark, J.M. Roesset, *Nonlinear Dynamic Response of Reinforced Concrete Frames*, Publication R76-38, Department of Civil Engineering, M.I.T, 1976.
- [17] E. Spacone, F.C. Filippou, F.F. Taucer, Fibre beam-column model for non-linear analysis of R/C frames: Part I. Formulation, *Earthquake Engineering and Structural Dynamics*, **25**, 711-725, 1996.
- [18] P. Kotronis, J. Mazars, Simplified modelling strategies to simulate the dynamic behaviour of RC walls. *Journal of Earthquake Engineering*, **9(2)**, 285-306, 2005.
- [19] C. Zeris, S.A. Mahin, Analysis of reinforced concrete beam-columns under uniaxial excitation, *Journal of Structural Engineering*, **114(4)**, 804-820, 1988.
- [20] A.M. Ismail, M.F.M. Fahmy, Z. Wu, Simulating the lateral performance of FRP-confined RC circular columns using a new eccentric-based stress-strain model. *Composite Structures*, **180**, 88-104, 2017.
- [21] M. Chellapandian, S. Suriya Prakash, A. Rajagopal, Analytical and finite element studies on hybrid FRP strengthened RC column elements under axial and eccentric compression. *Composite Structures*, **184**, 234-248, 2018.
- [22] A. Hain, A.E. Zaghi, M. Saiid Saiidi, Flexural behavior of hybrid concrete-filled fiber reinforced polymer tube columns. *Composite Structures*, **230**, 111540, 2019.

- 
- [23] N.M. Newmark, A method of computation for structural dynamics, *Journal of Engineering Mechanics*, **85(3)**, 67-94, 1959.
- [24] E.L. Wilson, *A Computer Program for the Dynamic Stress Analysis of Underground Structures*, SESM Report No.68-1, Division of Structural Engineering Structural Mechanics, University of California, 1968.
- [25] H.M. Hilber, T.J.R. Hughes, R.L. Taylor, Improved numerical dissipation for time integration algorithms in structural dynamics. *Earthquake Engineering and Structural Dynamics*, **25**, 283-292, 1977.
- [26] A.K. Chopra, *Dynamics of structures: theory and applications to earthquake engineering*, Fifth edition, Hoboken, NJ: Pearson, 2017.
- [27] J.E. Jackson, *A User's Guide To Principal Components*, Wiley series in probability and mathematical statistics, 1991.
- [28] Center of Engineering for Strong Motion Data CESMD ([www.strongmotioncenter.org](http://www.strongmotioncenter.org))
- [29] J.B. Mander, M.J.N. Priestley, R. Park, Theoretical stress-strain model for confined concrete, *Journal of Structural Engineering*, **114(8)**, 1804-1826, 1988.
- [30] F. Mazza, R. Labernarda, Effects of near-fault acceleration and non-acceleration pulses on pounding between in-plan irregular fixed-base and base-isolated buildings. *Structural Control and Health Monitoring*, e2992, 2022.