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Simultaneous interval state and fault estimation for continuous-time switched systems

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Abstract: This paper deals with simultaneous interval state and fault estimation for continuous-time switched systems subject to unknown but bounded state disturbances. By taking the actuator fault vector as a part of an extended state vector, the original system is transformed into an augmented descriptor system. Under this augmented descriptor form, and by using a new structure of interval observers, the conservatism of observer gain matrices can be significantly reduced. The effect of disturbances is attenuated based on an L_∞ approach. Then, the interval observer gains are computed by solving Linear Matrix Inequalities (LMIs) derived from a common Lyapunov function. Finally, a simulation example is given to illustrate the effectiveness of the proposed method.

Keywords: state and fault estimation, continuous-time switched systems, interval observers, descriptor system, L_∞ approach.

1. INTRODUCTION

With the continuous growing complexity of industrial systems and rising demand for higher performance and safety, the implementation of optimized fault estimation strategies has become a priority for many industrials in order to provide further accurate information of the fault such as magnitude, location and duration, (Gao et al. (2007); Zhang et al. (2013)) and to avoid severe and irreversible damage to human operator and equipment. In the past decades, several techniques have been introduced for various systems in the field of fault estimation such as in Han et al. (2018); Wang et al. (2021).

Many of the dynamical systems encountered in practice as mechanical, embedded and robotic, are characterized by hybrid processes which exhibit both discrete and continuous dynamics. Switched system is one of the most important complex systems, (Liberzon and Morse (1999)), and plays an essential role in many engineering applications. The problem of fault estimation for switched systems has attracted the interest of many researchers, (Du et al. (2012); Du and Jiang (2016); Li et al. (2019)). For instance, in Du et al. (2012), the problem of sensor fault estimation is investigated for time-delay switched systems based on a switched descriptor observer approach. The error system is robustly asymptotically stable and satisfies the H_∞ performance index. The proposed fault estimation algorithm in Du and Jiang (2016) is achieved for a discrete-time switched system with actuator fault and state delay using reduced-order observer and switched Lyapunov functions. More recently, the main purpose in Li et al. (2019) is to study the fault detection and estimation problems for switched systems based on DC–DC converters using sliding mode observers.

Many model-based fault estimation methods usually assume that the disturbances have a known probability distribution. However, from a practical point of view, it is not trivial to obtain the priori information of probability distribution. In this paper, the proposed technique is based on interval methods which only assumes that the disturbances are unknown but bounded, which is a more realistic assumption simple to implement in practical systems. In the literature, a considerable attention has been paid to the study of interval state and fault estimation for dynamical systems, (Guo and Zhu (2017); Zhang et al. (2020); Tian et al. (2020); Marouani et al. (2021)). In Guo and Zhu (2017), a fault detection and interval reconstruction scheme are introduced based on interval observers for discrete-time systems subject to both actuator faults and disturbances. Authors in Zhang et al. (2020) propose a state augmentation technique to design interval fault estimation for descriptor discrete-time systems with unknown but bounded disturbances and measurement noises. The main idea is to transform the original system into an augmented descriptor system by considering the fault vector as an auxiliary state. It helps to estimate simultaneously the original state and fault. In Tian et al. (2020), fault estimation is investigated based on both interval and unknown input observers for high-speed railway traction motor subject to sensor fault and disturbances. The main purpose in Marouani et al. (2021) is to design a simultaneous input and state interval observer for discrete-time linear switched systems. The effect of unknown disturbances and measurement noise is reduced according to an H_∞ formalism.

Motivated by the above discussions, most of interval fault estimation are designed in a discrete-time settings. To the best of the authors' knowledge, robust interval fault and state estimation has not been fully investigated for switched systems. This study has three main contributions:

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(1) First, a new interval observer is presented to achieve simultaneously the state and fault estimation for a class of uncertain continuous-time linear switched systems.
(2) Second, the proposed observer structure allows reducing the conservatism of gain matrices and offers more degrees of design freedom by integrating weighted matrices in the structure of the observer design.
(3) Finally, the designed technique can attenuate the effect of state disturbances by incorporating L_∞ performance in order to obtain accurate interval estimation results.

The remainder of this paper is structured as follows. Notations and some preliminaries are introduced in Section 2. Section 3 presents the problem statement. Main results are described in Section 4. In Section 5, simulation results are shown to illustrate the effectiveness of the proposed methods. Finally, the paper is concluded in Section 6.

2. PRELIMINARIES

2.1 Notations

$\mathbb{R}$, $\mathbb{R}^n$, $\mathbb{R}^{m\times n}$ and $\mathbb{Z}_+$ represent respectively the sets of real numbers, n dimensional, $m\times n$ dimensional Euclidean space and the set of non-negative integers. $\|x\|$ denotes the Euclidean norm of a signal $x\in\mathbb{R}^n$. The L_∞ norm of a signal x is defined as $\|x\|_\infty = \sup_{t\geq 0}\|x(t)\|$. $\mathscr{I} = \overline{1,N}, N\in\mathbb{Z}_+$ is the set of non-negative integers $\{1,...,N\}$. $(*)$ represents the terms introduced by symmetry. I_n denotes the identity matrix with n dimensions. For a matrix A, $A\succ 0$ means that A is positive definite, $A^\dagger$ represents its pseudo-inverse, $A^+ = \max\{0,A\}$, $A^- = A^+ - A$ and $|A| = A^+ + A^-$. Throughout this paper, the inequalities $\leq$, $\geq$, $<$ and $>$ should be interpreted elementwise.

2.2 Switched linear system

Let us define a switched linear system (SLS) with the following modeling:

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + D_{\sigma(t)}w(t) + F_{\sigma(t)}f(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

$x(t)\in\mathbb{R}^{n_x}$ is the continuous state of the system, $u\in\mathbb{R}^{n_u}$ is the known input, $y(t)\in\mathbb{R}^{n_y}$ is the output, $w(t)\in\mathbb{R}^{n_w}$ is an unknown input which is assumed to be bounded, $f(t)\in\mathbb{R}^{n_f}$ represents an additive fault. The SLS is defined by N subsystems. The switching signal $\sigma(t):\mathbb{R}^+\to\mathscr{I}=\{1,2,\ldots,N\}$ is a piecewise right-continuous function which allows selecting the active subsystem $q=\sigma(t)$ in continuous time. In this paper, the fault f and the continuous state x are assumed to be unknown.

3. PROBLEM STATEMENT

3.1 Interval observer design

In this section, the aim is to propose an interval observer to deal with simultaneous fault and state estimation in the presence of unknown state disturbances. The gain matrices are computed such that the closed-loop system is stable according to an L_∞ index. To proceed, some assumptions, lemmas and definitions are presented.

Assumption 1. Assume that the state disturbance w is unknown but bounded with a priori known bounds such that

$$\underline{w} \leq w \leq \overline{w}$$

where $\underline{w},\overline{w}\in\mathbb{R}^{n_w}$.

Assumption 2. Assume that the derivative of the actuator fault vector is bounded with a priori known bounds such that

$$\underline{\dot{f}} \leq \dot{f} \leq \overline{\dot{f}}$$

where $\underline{\dot{f}},\overline{\dot{f}}\in\mathbb{R}^{n_f}$.

Remark 1. It is worth noting that :

- Assumption 2 is commonly used in the literature. For several practical systems, faults usually need some time to establish themselves, and then they remain somehow constant, which implies that their derivatives are energy bounded. The same assumption can be found in other previous works (Rodrigues et al. (2014); Li et al. (2018); Mu et al. (2019)).
- Assumption 2 is essential when the fault signal changes frequently.

Assumption 3. The upper and lower bounds of the initial state and fault are chosen such that:

$$\begin{aligned} \underline{x}(0) &\leq x(0) \leq \overline{x}(0) \\ \underline{f}(0) &\leq f(0) \leq \overline{f}(0) \end{aligned}$$

Lemma 1. Given matrices $A\in\mathbb{R}^{a\times b}$, $B\in\mathbb{R}^{b\times c}$ and $C\in\mathbb{R}^{a\times c}$, if $\text{rank}(B)=c$, then the general solution of the following equation $AB=C$ is given by

$$A = CB^\dagger + S(I - BB^\dagger)$$

where $S\in\mathbb{R}^{a\times b}$ is an arbitrary matrix.

Lemma 2. (Efimov et al. (2012)) Let $A\in\mathbb{R}^{m\times n}$ be a constant matrix and $x\in\mathbb{R}^n$ be a vector such that $\underline{x}\leq x\leq\overline{x}$, thus

$$A^+\underline{x} - A^-\overline{x} \leq Ax \leq A^+\overline{x} - A^-\underline{x}.$$

Lemma 3. (Rami et al. (2008)) Consider the system described by:

$$\dot{x}(t) = Ax(t) + u(t) \tag{2}$$

If A is Metzler, the input u verifies $u(t)\geq 0$ and the initial condition $x(0)$ is chosen as $x(0)\geq 0$, then the state x stays nonnegative for all $t\geq 0$. The system (2) is said cooperative.

Definition 1. A matrix $A\in\mathbb{R}^{n\times n}$ is called Metzler if there exists $\varepsilon\in\mathbb{R}^+$ such that:

$$A + \varepsilon I_n \geq 0 \tag{3}$$

3.2 Descriptor switched linear system

The system (1) is firstly formulated as an augmented descriptor system representation by considering the actuator fault $f(t)$ as an auxiliary state vector, i.e,

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ f(t) \end{bmatrix}$$

The new augmented system is described with the following equations :

$$\begin{aligned} \tilde{E}\dot{\tilde{x}}(t) &= \tilde{A}_q\tilde{x}(t) + \tilde{B}_q u(t) + \tilde{D}_q w(t) + \tilde{G}\dot{f}(t) \\ y(t) &= \tilde{C}\tilde{x}(t) \end{aligned} \tag{4}$$

where the matrices $\tilde{E}$, $\tilde{A}_q$, $\tilde{B}_q$, $\tilde{D}_q$ and $\tilde{C}$ are given by

$$\tilde{E} = \begin{bmatrix} I_{n_x} & 0 \\ 0 & I_{n_f} \end{bmatrix}, \tilde{A}_q = \begin{bmatrix} A_q & F_q \\ 0 & 0 \end{bmatrix}, \tilde{B}_q = \begin{bmatrix} B_q \\ 0 \end{bmatrix}$$

$$\tilde{D}_q = \begin{bmatrix} D_q \\ 0 \end{bmatrix}, \tilde{G} = \begin{bmatrix} 0 \\ I_{n_f} \end{bmatrix}, \tilde{C} = [\, C \ 0 \,]$$

Note that, the interval estimation problem for (1) is converted to an interval observer design for the augmented descriptor switched system (4) to estimate simultaneously the state vector $x(t)$ and the actuator fault $f(t)$.

Remark 2. There is no theoretical issue if a sensor fault f_s is added in the output equation of (1). In this case, we consider the following augmented state $\tilde{x}$ defined by :

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ f(t) \\ f_s(t) \end{bmatrix}$$

4. SIMULTANEOUS INTERVAL FAULT AND CONTINUOUS STATE ESTIMATION FOR DESCRIPTOR SWITCHED SYSTEM

4.1 Interval observer structure

To design the interval observer, two properties have to be satisfied: (i) the cooperativity property which consists in finding intervals to the augmented state vector such that $\underline{\tilde{x}}(t) \le \tilde{x}(t) \le \overline{\tilde{x}}(t)$, (ii) stability and robustness properties which care the convergence and the tightness of estimated intervals. To this end, observer gains L_q need to be chosen such that the matrices $\tilde{A}_q - L_q\tilde{C}$ are Metzler and the estimation errors are stable and robust against uncertainties, which is usually restrictive. One can think about finding a nonsingular transformation such that $P(\tilde{A}_q - L_q\tilde{C})P^{-1}$ are metzler, (Mazenc and Bernard (2011); Raïssi et al. (2011)). Nevertheless, the existence of a common transformation P is not always trivial. Motivated by (Zammali et al. (2021)), we propose the following interval observer:

$$\begin{array}{rcl}\dot{\overline{\xi}}(t) &=& T_q\tilde{A}_q\overline{\tilde{x}}(t) + T_q\tilde{B}_q u(t) + \overline{\Delta} + \\ && L_q(y(t) - \tilde{C}\overline{\tilde{x}}(t)) + T_q\tilde{G}\dot{\overline{f}}(t) \\ \overline{\tilde{x}}(t) &=& \overline{\xi}(t) + N_q y(t) \\ \dot{\underline{\xi}}(t) &=& T_q\tilde{A}_q\underline{\tilde{x}}(t) + T_q\tilde{B}_q u(t) + \underline{\Delta} + \\ && L_q(y(t) - \tilde{C}\underline{\tilde{x}}(t)) + T_q\tilde{G}\dot{\underline{f}}(t) \\ \underline{\tilde{x}}(t) &=& \underline{\xi}(t) + N_q y(t) \\ \overline{f}(t) &=& C_f^+\overline{\tilde{x}}(t) - C_f^-\underline{\tilde{x}}(t) \\ \underline{f}(t) &=& C_f^+\underline{\tilde{x}}(t) - C_f^-\overline{\tilde{x}}(t)\end{array} \tag{5}$$

where $\overline{\xi}(t)$, $\underline{\xi}(t)$ denote intermediate variables, $\overline{\tilde{x}}(t)$, $\underline{\tilde{x}}(t)$ are the upper and lower bounds of $\tilde{x}(t)$ respectively. $\overline{\Delta}$ and $\underline{\Delta}$ are given using Lemma 2:

$$\begin{cases}\overline{\Delta} = (T_q\tilde{D}_q)^+\overline{w} - (T_q\tilde{D}_q)^-\underline{w} \\ \underline{\Delta} = (T_q\tilde{D}_q)^+\underline{w} - (T_q\tilde{D}_q)^-\overline{w}\end{cases}$$

C_f is given by $C_f = [0 \quad I_{n_f}]$. L_q are the observer gains. T_q and N_q are constant matrices that should be designed to satisfy

$$T_q\tilde{E} + N_q\tilde{C} = I_{n_{\tilde{x}}} \tag{6}$$

Based on Lemma 1, the general solution of (6) is given by

$$[T_q \ N_q] = \begin{bmatrix}\tilde{E} \\ \tilde{C}\end{bmatrix}^\dagger + S_q\left(I_{n_{\tilde{x}}+n_y} - \begin{bmatrix}\tilde{E} \\ \tilde{C}\end{bmatrix}\begin{bmatrix}\tilde{E} \\ \tilde{C}\end{bmatrix}^\dagger\right) \tag{7}$$

where $S_q \in \mathbb{R}^{n_{\tilde{x}} \times (n_{\tilde{x}}+n_y)}$ are arbitrary matrices which will be designed such that all matrices T_q are of full rank.

Remark 3. Compared with the classical interval observer (Efimov et al. (2012)), the proposed one (5) can reduce the conservatism of gain matrices and provide more degree of freedom by introducing the weighted matrices T_q and N_q. The meanings of T_q and N_q are selecting information from model and measurement outputs, respectively. According to the equation constraints in (6), the information from model and measurements is combined together for the augmented state estimation and no new error expression is presented at the same time.

Remark 4. Note that in the case of constant or slow time varying faults, the following interval observer

$$\begin{array}{rcl}\dot{\overline{\xi}}(t) &=& T_q\tilde{A}_q\overline{\tilde{x}}(t) + T_q\tilde{B}_q u(t) + \overline{\Delta} + \\ && L_q(y(t) - \tilde{C}\overline{\tilde{x}}(t)) \\ \dot{\underline{\xi}}(t) &=& T_q\tilde{A}_q\underline{\tilde{x}}(t) + T_q\tilde{B}_q u(t) + \underline{\Delta} + \\ && L_q(y(t) - \tilde{C}\underline{\tilde{x}}(t)) \\ \overline{\tilde{x}}(t) &=& \overline{\xi}(t) + \overline{N}_q y(t) \\ \underline{\tilde{x}}(t) &=& \underline{\xi}(t) + \underline{N}_q y(t)\end{array}, \tag{8}$$

may achieve a satisfied estimation performance.

4.2 Framer design and stability analysis

In this subsection, we aim to :

(1) calculate two estimates $\underline{\tilde{x}}$ and $\overline{\tilde{x}}$ such that

$$\underline{\tilde{x}}(t) \le \tilde{x}(t) \le \overline{\tilde{x}}(t) \tag{9}$$

(2) compute the observer gains L_q and the weighted matrices T_q and N_q by using the L_∞ formalism in order to improve the accuracy of the interval framers.

Based on the proposed structure of the observer (5) and by combining (4) and (6), $\dot{\tilde{x}}$ can be written as follows:

$$\begin{array}{rcl}\dot{\tilde{x}}(t) &=& (T_q\tilde{E} + N_q\tilde{C})\dot{\tilde{x}}(t) \\ &=& T_q\tilde{A}_q\tilde{x}(t) + T_q\tilde{B}_q u(t) + T_q\tilde{D}_q w(t) + T_q\tilde{G}\dot{f}(t) + \\ && N_q\dot{y}(t)\end{array} \tag{10}$$

To synthesise the observer in (5), we introduce the following upper and lower estimation errors:

$$\begin{array}{rcl}\overline{e}(t) &=& \overline{\tilde{x}}(t) - \tilde{x}(t) \\ \underline{e}(t) &=& \tilde{x}(t) - \underline{\tilde{x}}(t)\end{array} \tag{11}$$

According to (5) and (10), the error dynamics are obtained as

$$\begin{array}{rcl}\dot{\overline{e}}(t) &=& (T_q\tilde{A}_q - L_q\tilde{C})\overline{e}(t) + \overline{\Delta} - T_q\tilde{D}_q w(t) + \\ && T_q\tilde{G}\dot{\overline{f}}(t) - T_q\tilde{G}\dot{f}(t) \\ \dot{\underline{e}}(t) &=& (T_q\tilde{A}_q - L_q\tilde{C})\underline{e}(t) - \underline{\Delta} + T_q\tilde{D}_q w(t) - \\ && T_q\tilde{G}\dot{\underline{f}}(t) + T_q\tilde{G}\dot{f}(t)\end{array} \tag{12}$$

Theorem 1. **(Framer design)** For system (4), let Assumptions 1, 2 and 3 hold, $\overline{\tilde{x}}(t)$ and $\underline{\tilde{x}}(t)$ in (5) satisfy the inclusion

$$\underline{\tilde{x}}(t) \le \tilde{x}(t) \le \overline{\tilde{x}}(t)$$

if $T_q\tilde{A}_q - L_q\tilde{C}$ are Metzler for all $t \ge 0$ and $\underline{\tilde{x}}(0)$, $\overline{\tilde{x}}(0)$ are chosen such that $\underline{\tilde{x}}(0) \le \tilde{x}(0) \le \overline{\tilde{x}}(0)$.

Proof.
According to Assumptions 1 and 2 , we have

$$\begin{array}{r}\overline{\Delta} - T_q\tilde{D}_q w(t) + T_q\tilde{G}\dot{\overline{f}}(t) - T_q\tilde{G}\dot{f}(t) \ \ge 0 \\ -\underline{\Delta} + T_q\tilde{D}_q w(t) - T_q\tilde{G}\dot{\underline{f}}(t) + T_q\tilde{G}\dot{f}(t) \ \ge 0\end{array}$$

In addition, if Assumption 3 is satisfied, then, $\overline{e}(0) \ge 0$ and $\underline{e}(0) \ge 0$. Applying Lemma 3 to (12), the inclusion

$$\underline{\tilde{x}}(t) \le \tilde{x}(t) \le \overline{\tilde{x}}(t)$$

holds for all $t \ge 0$ if $T_q\tilde{A}_q - L_q\tilde{C}$ are Metzler.
End Proof.

In order to study the stability of (12), we propose to study the stability of $\dot{\zeta}(t) = \dot{\bar{e}}(t) + \dot{\underline{e}}(t)$. According to (12), $\dot{\zeta}(t)$ can be deduced as :

$$\dot{\zeta}(t) = \mathscr{A}_q \zeta(t) + \mathscr{W}(t) + \mathscr{F}(t) \tag{13}$$

where

$$\begin{cases} \mathscr{A}_q = T_q\tilde{A}_q - L_q\tilde{C},\ \mathscr{F} = T_q\tilde{G}\dot{\bar{f}} - T_q\tilde{G}\dot{\underline{f}} \\ \mathscr{W} = \overline{\Delta} - \underline{\Delta} \end{cases}$$

Based on the error dynamic equations given in (13), the objective is to design the proposed interval observer in (5) such that the error system is stable and the effect of the uncertainties is reduced. For this end, the L_∞ technique is used such that for a given scalar $\gamma > 0$, the error signal should satisfy the following inequality

$$\|\zeta\| < \sqrt{\gamma^2(\tilde{\mathscr{W}}^2 + \tilde{\mathscr{F}}^2)) + \gamma\lambda V(0)e^{-\lambda t}}$$

where $\lambda > 0$, $V(0) = \zeta(0)^T P\zeta(0)$, $P \succ 0 \in \mathbb{R}^{n_{\tilde{x}} \times n_{\tilde{x}}}$, $\tilde{\mathscr{W}}$ and $\tilde{\mathscr{F}}$ are known constants such that $\tilde{\mathscr{W}} = \|\mathscr{W}\|_\infty$ and $\tilde{\mathscr{F}} = \|\mathscr{F}\|_\infty$. The following theorem is introduced to design the observer gains L_q and the weighted matrices T_q and N_q.

Theorem 2. **Gains computation** Let Assumption 3 hold. Given scalars $\lambda > 0$ and $\varepsilon > 0$ if there exist scalars $\gamma > 0$ and $\mu > 0$, a positive definite diagonal matrix, $P = P^T \succ 0$, $P \in \mathbb{R}^{n_{\tilde{x}} \times n_{\tilde{x}}}$ and constant matrices $Q_q \in \mathbb{R}^{n_{\tilde{x}} \times n_y}$ and $Y_q \in \mathbb{R}^{n_{\tilde{x}} \times (n_{\tilde{x}}+n_y)}$ such that:

$$P\Theta^\dagger \alpha_1 \tilde{A}_q + Y_q \Psi \alpha_1 \tilde{A}_q - Q_q C + \varepsilon P \geq 0. \tag{14}$$

$$\begin{bmatrix} \mathscr{A}_q^T P + P\mathscr{A}_q + \lambda P & P & P \\ (*) & -\mu I_{n_{\mathscr{W}}} & 0 \\ (*) & (*) & -\mu I_{n_{\mathscr{F}}} \end{bmatrix} \prec 0. \tag{15}$$

$$\begin{bmatrix} \lambda P & 0 & 0 & I_{n_\zeta} \\ (*) & (\gamma-\mu)I_{n_{\mathscr{W}}} & 0 & 0 \\ (*) & (*) & (\gamma-\mu)I_{n_{\mathscr{F}}} & 0 \\ (*) & (*) & (*) & \gamma I_{n_\zeta} \end{bmatrix} \succ 0. \tag{16}$$

then, the error dynamics system in (13) is bounded as

$$\|\zeta\| < \sqrt{\gamma^2(\tilde{\mathscr{W}}^2 + \tilde{\mathscr{F}}^2)) + \gamma\lambda V(0)e^{-\lambda t}} \tag{17}$$

where $V(0) = \zeta(0)^T P\zeta(0)$. Moreover, the observer gains L_q, the weighted matrices T_q, and N_q are given by :

$$\begin{cases} L_q = P^{-1}Q_q \\ T_q = \Theta^\dagger \alpha_1 + P^{-1} Y_q \Psi \alpha_1 \\ N_q = \Theta^\dagger \alpha_2 + P^{-1} Y_q \Psi \alpha_2 \end{cases} \tag{18}$$

α_1, α_2, Θ and Ψ are given by

$$\alpha_1 = \begin{bmatrix} I_{n_{\tilde{x}}} \\ 0 \end{bmatrix},\ \alpha_2 = \begin{bmatrix} 0 \\ I_{n_y} \end{bmatrix},\ \Theta = \begin{bmatrix} I_{n_{\tilde{x}}} \\ C \end{bmatrix},\ \Psi = I_{n_{\tilde{x}}+n_y} - \Theta\Theta^\dagger$$

Proof.

First of all, the Metzler property of the matrix $\mathscr{A}_q$ is satisfied using Definition 1. In fact, if there exists $\varepsilon \in \mathbb{R}_+$ such that $\mathscr{A}_q + \varepsilon I_n \geq 0$, thus, $P\mathscr{A}_q + \varepsilon P \geq 0$. By replacing T_q by its expression in (18), the inequality (14) is satisfied.

Now, Define a Lyapunov function $V(\zeta) = \zeta(t)^T P\zeta(t)$, $P = P^T \succ 0$. By pre and post-multiplying the matrix inequality in (15) with $[\zeta(t)^T \quad \mathscr{W}^T(t) \quad \mathscr{F}^T(t)]$ and its transpose, respectively, we have

$$\begin{bmatrix} \zeta(t) \\ \mathscr{W}(t) \\ \mathscr{F}(t) \end{bmatrix}^T \begin{bmatrix} \mathscr{A}_q^T P + P\mathscr{A}_q + \lambda P & P & P \\ P^T & -\mu I_{n_{\mathscr{W}}} & 0 \\ P^T & 0 & -\mu I_{n_{\mathscr{F}}} \end{bmatrix} \begin{bmatrix} \zeta(t) \\ \mathscr{W}(t) \\ \mathscr{F}(t) \end{bmatrix} \prec 0$$

which implies that

$$\dot{V}(t) + \lambda V(t) < \mu(\mathscr{W}^T\mathscr{W} + \mathscr{F}^T\mathscr{F}) \tag{19}$$

Integrating the inequality (19) over the interval $[0,t]$, we obtain

$$V(t) < \frac{\mu}{\lambda}(\tilde{\mathscr{W}}^2 + \tilde{\mathscr{F}}^2) + V(0)e^{-\lambda t} \tag{20}$$

where $\tilde{\mathscr{W}}$ and $\tilde{\mathscr{F}}$ are known constants given by $\tilde{\mathscr{W}} = \|\mathscr{W}\|_\infty$ and $\tilde{\mathscr{F}} = \|\mathscr{F}\|_\infty$.

In addition, using the Schur complement lemma, (16) is equivalent to

$$\begin{bmatrix} \lambda P & 0 & 0 \\ (*) & (\gamma-\mu)I_{n_{\mathscr{W}}} & 0 \\ (*) & (*) & (\gamma-\mu)I_{n_{\mathscr{F}}} \end{bmatrix} - \frac{1}{\gamma}\begin{bmatrix} I_{n_\zeta} \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} I_{n_\zeta} & 0 & 0 \end{bmatrix} \succ 0. \tag{21}$$

Then, pre-multiplying and post-multiplying (21) with $[\zeta(t)^T \ \mathscr{W}^T(t) \ \mathscr{F}^T(t)]$ and its transpose, respectively, one can get

$$\|\zeta\|^2 < \gamma(\lambda V(t) + (\gamma-\mu)(\tilde{\mathscr{W}}^2 + \tilde{\mathscr{F}}^2)) \tag{22}$$

Substituting (20) into (22) yields

$$\|\zeta\|^2 < \gamma^2(\tilde{\mathscr{W}}^2 + \tilde{\mathscr{F}}^2) + \gamma\lambda V(0)e^{-\lambda t}$$

which follows

$$\|\zeta\| < \sqrt{\gamma^2(\tilde{\mathscr{W}}^2 + \tilde{\mathscr{F}}^2)) + \gamma\lambda V(0)e^{-\lambda t}}$$

Therefore, the convergence of the proposed interval observer and the L_∞ criterion in (17) are satisfied.

The value of γ could be minimised as follows:

$$\min \gamma \quad s.t. \quad (16) \tag{23}$$

It is worth noting that in some cases, optimising the steady-state gains may lead to a conservative initial estimation. Fortunately, as shown in (17), one can remark that the bound of the estimation error ζ is affected by $V(0)$, which is a function of the initial estimation error and which exponentially tend to zero. Consequently, the conservatism related to the initial error will drop at an exponential rate.

End Proof.

Remark 5. In this paper, the stability analysis is studied by applying common Lyapunov function. The results given by Theorem 2 can be extended using Multiple Lyapunov functions under an ADT switching signal.

5. NUMERICAL EXAMPLE

In this section, we provide a simulation example to illustrate the feasibility of the proposed estimation method. A continuous-time switched system (1) is defined as three subsystems, $N = 3$, with:

$$A_1 = \begin{bmatrix} -0.6 & 0.5 & -1 \\ -0.2 & -0.5 & -1 \\ 0 & -0.2 & -0.5 \end{bmatrix},\ A_2 = \begin{bmatrix} -0.9 & 0.8 & 0.5 \\ -2 & 0.5 & 1 \\ 0 & 0.5 & -0.1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -0.80.90.4 \\ -1.50.40.9 \\ 0.1-0.4-0.1 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 \\ 0.1 \\ 1.3 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 0 \\ 1 \\ 0.5 \end{bmatrix},\ D_1 = \begin{bmatrix} 0.02 \\ 0.01 \\ 0.01 \end{bmatrix},\ D_2 = \begin{bmatrix} 0.05 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$D_3 = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.01 \end{bmatrix} \quad F_1 = \begin{bmatrix} -0.5 \\ 0.1 \\ -0.1 \end{bmatrix} \quad F_2 = \begin{bmatrix} -0.4 \\ 0.2 \\ -1 \end{bmatrix}$$

$$F_3 = \begin{bmatrix} 0.5 \\ -2 \\ -0.5 \end{bmatrix} \quad C = \begin{bmatrix} 1.2 & 0.01 & 0 \\ 0.1 & 1.1 & 0.1 \end{bmatrix}$$

In this example, $w(t) \in \mathbb{R}$ is a uniformly distributed bounded signal such that $|w(t)| \leq 0.1$. The state initial conditions are set as $\tilde{x}(0) = [0 \quad 0 \quad 0 \quad 0]^T$, $\underline{\tilde{x}}(0) = [-0.5 \quad -0.5 \quad -0.5 \quad -0.01]^T$ and $\overline{\tilde{x}}(0) = [0.5 \quad 0.5 \quad 0.5 \quad 0.01]^T$ such that $\underline{\tilde{x}}(0) \leq \tilde{x}(0) \leq \overline{\tilde{x}}(0)$. Figure 1 shows the evolution of the switching signal. It indicates the active mode of the continuous-time switched system.

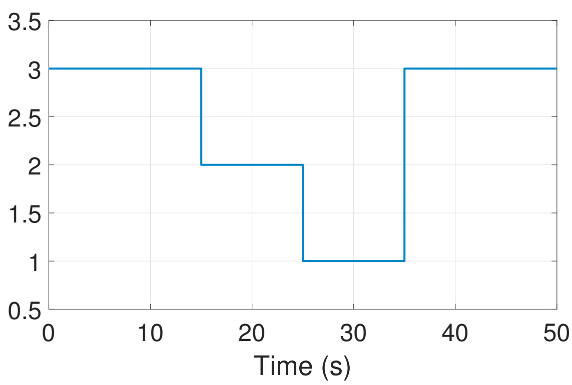

Fig. 1. Evolution of the switching signal

Based on the TNL structure used in (5), the following Lyapunov matrix P is given by:

$$P = \begin{bmatrix} 1.1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 7.07 & 0 \\ 0 & 0 & 0 & 14.9 \end{bmatrix}$$

The proposed interval observer is designed with $\lambda = 0.5$ and $\varepsilon = 0.5$. By solving LMIs in Theorem 2, $\gamma = 2.914$, $\mu = 8.49$ and the observer gains L_q can be obtained as follows:

$$L_1 = \begin{bmatrix} 4.22 & 0.05 \\ -0.31 & 4.71 \\ -0.03 & -0.53 \\ -1.37 & 3.21 \end{bmatrix}, \; L_2 = \begin{bmatrix} -0.57 & 0.24 \\ -5.94 & 5.29 \\ 8.23 & -4.11 \\ -0.38 & 2.33 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 0.33 & 2.37 \\ -5.13 & 7.76 \\ 4.81 & -4.33 \\ -1.06 & -0.58 \end{bmatrix}$$

For simulation purposes, two types of actuator faults are created as follows:

(1) Time-varying fault

$$f(t) = \begin{cases} 0.01 sin(t) & 10 \leq t \leq 40 \\ 0 & \text{otherwise} \end{cases}$$

(2) Abrupt sloped fault

$$f(t) = \begin{cases} 0.1t - 1 & 10 \leq t \leq 15 \\ -0.1t + 4 & 35 \leq t \leq 40 \\ 0.5 & 15 \leq t \leq 35 \\ 0 & \text{otherwise} \end{cases}$$

Simulation results are given in Figs. 2-5 where the red and blue dashed lines correspond respectively to the estimated upper and lower bounds of the actuator faults and the continuous state.

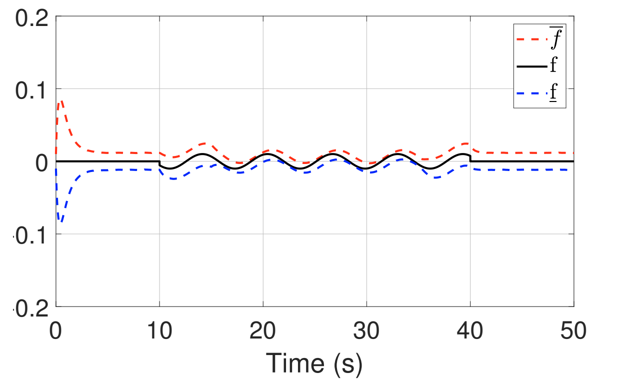

Fig. 2. Evolution of the time-varying fault and its estimated bounds

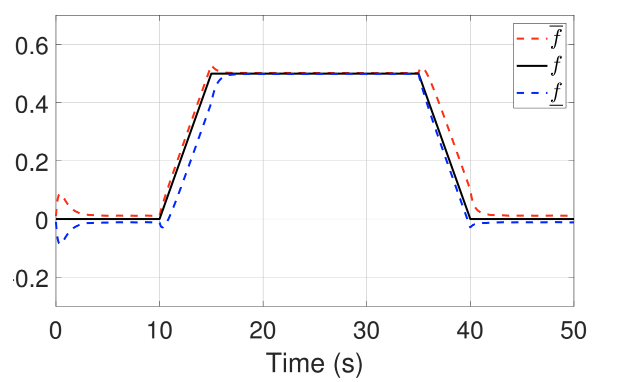

Fig. 3. Evolution of the abrupt sloped fault and its estimated bounds

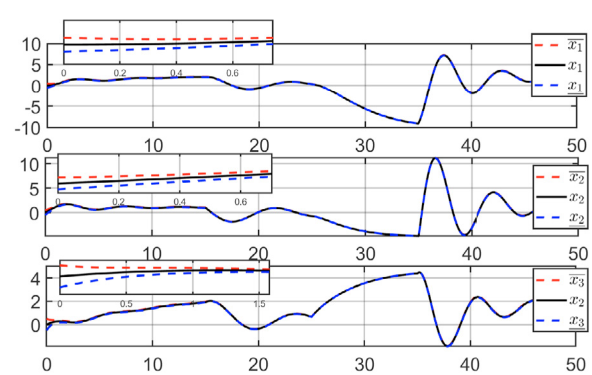

Fig. 4. Evolution of the state and its estimated bounds under the time-varying fault

These results show that, the designed observer can effectively estimate the framers of continuous system states and faults despite the presence of unknown state disturbances. In addition, to show that the proposed method provides an accurate estimation, a comparison has been made with the fault estimation approach without considering robustness analysis in the case of time-varying faults.

It is clear that the estimated framers using the proposed method are more accurate. This is reasonable since in the designed

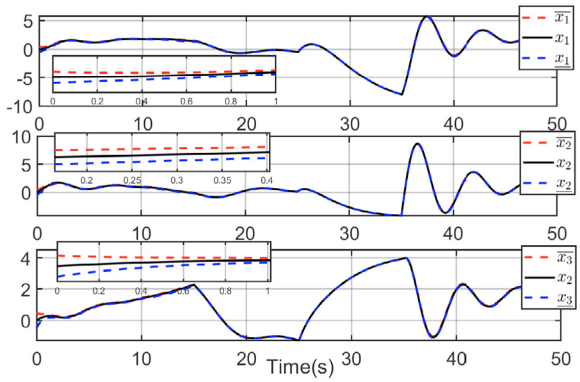

Fig. 5. Evolution of the state and its estimated bounds under the abrupt sloped fault

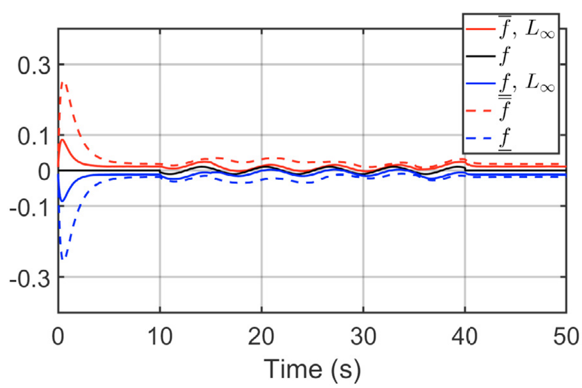

Fig. 6. Comparison between the results of fault estimation obtained with and without L_∞ optimisation

approach, the impact of uncertainties is attenuated using L_∞ performances.

6. CONCLUSION

This paper investigates the problem of fault estimation for a class of continuous-time switched systems affected by actuator faults and subject to unknown but bounded state disturbances. By using state augmentation technique, an interval observer is designed to estimate simultaneously system states and actuator faults. Then, an L_∞ technique is introduced to improve the estimation accuracy. Finally, the simulation results illustrate the efficiency of the proposed interval method. For future works, extensions of these results with LPV switched systems will be considered.

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