Fractional-Order System: Control Theory and Applications

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fractal and fractional

Editorial

Fractional-Order System: Control Theory and Applications

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(Fractional) differential equations have seen increasing use in physics, signal processing, fluid mechanics, viscoelasticity, mathematical biology, electrochemistry, and many other fields over the last two decades, providing a new and more realistic way to capture memory-dependent phenomena and irregularities inside systems using more sophisticated mathematical analysis (see, for example, [1] and the references therein).

The study of the stability of (fractional) differential equations has attracted a lot of attention as a result of its growing applications. Furthermore, fractional- and integer-order controllers have received increased attention in recent years. Among these are optimal control, CRONE controllers, fractional PID controllers, lead-lag compensators, and sliding mode control.

The purpose of this Special Issue is to carry out studies on fractional/integer-order control theory and its applications to practical systems modeled using fractional/integer-order differential equations such as design, implementation, and application of fractional/integer-order control to electrical circuits and systems, mechanical systems, chemical systems, biological systems, finance systems, etc.

Ten high-quality papers were accepted for publication in this Special Issue. The papers were written by different authors (note that no author published more than one paper, which proves the wide scope of the Special Issue). The published papers are briefly summarized as follows.

According to [2], the discrete fractional Fourier transform (DFRFT) has several definitions, the most common of which is the multiweighted fractional Fourier transform (M-WFRFT). It is difficult to demonstrate its unitarity. The weighted-type fractional Fourier transform, fractional-order matrix, and eigendecomposition-type fractional Fourier transform are used as basic functions to demonstrate and describe unitarity. They observed that the M-WFRFT has just four effective weighting terms, none of which are extended to M terms, as stated by the definition. Furthermore, the program code is examined, and the results demonstrate that the prior work (Digit Signal Process 2020: 104: 18) for unitary verification based on MATLAB is incorrect.

According to [3], there has been a recent surge in the number of papers addressing the overall issue of fractional-order controllers, with a concentration on fractional-order PID. This controller has been offered in several versions, each with its own set of tweaking techniques and implementation possibilities. A number of recent studies have discussed the practical application of such controllers. However, industrial acceptance of these controllers is still a long way off. Auto-tuning approaches for fractional-order PIDs may increase their desirability in relation to industrial applications. The existing auto-tuning approaches for fractional-order PIDs are reviewed in this work. The emphasis is on the most recent discoveries. For various processes, a comparison of many auto-tuning algorithms is addressed. Numerical examples are provided to demonstrate the applicability of the methodologies, which might be applied to simple industrial operations.

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The study [4] proposes an interval estimator for a fourth-order nonlinear susceptible–exposed–infected–recovered (SEIR) model with disturbances using noisy counts of susceptible patients given by Public Health Services (PHS). According to the World Health Organization, infectious diseases are the leading cause of mortality among the top 10 causes of death worldwide (WHO). As a result, tracking and assessing the progression of these diseases is critical for developing intervention methods. The authors investigate a real-world situation in which some uncertain variables, such as model disturbances and uncertain input and output measurement noise, are not precisely available but fall within an interval. Furthermore, the unclear transmission bound rate from the susceptible to the exposed stage cannot be measured. They created an interval estimator based on an observability matrix that yields a tight interval vector for the SEIR model's actual states in a guaranteed manner without computing the observer gain. The developed approach provides additional freedom because it is not dependent on observer gain. For the estimated state vector, the convergence of the width to a known value in a finite period is explored to demonstrate the stability of the estimation error. Finally, simulation results show that the suggested approach performs well.

Ref. [5] discusses a novel finite time stability (FTS) of neutral fractional-order systems with a time delay (NFOTSs). In light of this, the Gronwall inequality is used to demonstrate the FTSs of NFOTSs in the literature. The application of fixed-point theory to show the FTS of NFOTSs is a novel component of our proposed study. Finally, two instances are used to validate and substantiate the theoretical contributions.

The authors of [6] introduce a framework of distributed interval observers for fractional-order multiagent systems with nonlinearity. First, a frame was created to construct the system's upper and lower boundaries. The positivity of the error dynamics might be ensured by applying monotone system theory, implying that the constraints could trap the initial state. Second, a sufficient condition was used to ensure that distributed interval observers are bounded. The adequate condition was then based on an expansion of the Lyapunov function in the realm of fractional calculus. An algorithm related to the observer design technique was also provided. Finally, a numerical simulation was utilized to demonstrate the distributed interval observer's usefulness.

The paper [7] investigates an approximate method for solving the generalized fractional diffusion equation that combines the finite difference and collocation methods (GFDE). The presented method's convergence and stability analyses are also thoroughly established. To ensure the proposed method's effectiveness and accuracy, test examples with different scale and weight functions are taken into account, and the numerical results obtained are compared to the existing methods in the literature. The suggested method works particularly well with generalized fractional derivatives (GFDs), as the existence of scale and weight functions in a GFD makes discretization and further analysis problematic.

According to [8], autonomous underwater vehicles (AUVs) have a wide range of uses due to their capacity to travel great distances, their ability conceal themselves well, their high level of intelligence, and their ability to replace humans in dangerous missions. AUV motion control systems, which can assure steady operation in the complicated ocean environment, have piqued the interest of researchers. The authors suggest a single-input fractional-order fuzzy logic controller (SIFOFLC) as an AUV motion-control system in this research. First, a single-input fuzzy logic controller (SIFLC) based on the signed distance approach was presented, with its control input being a linear combination of the error signal and its derivative. The SIFLC reduces the controller design and calculation procedure significantly. Then, a SIFOFLC with the error signal's derivative extending to a fractional order was produced, providing additional flexibility and adaptability. Finally, comparative numerical simulations of spiral dive motion control were performed to validate the superiority of the suggested control algorithm. Meanwhile, the hybrid particle swarm optimization (HPSO) technique was used to optimize the parameters of several controllers. The simulation results demonstrate the suggested control algorithm's enhanced stability and transient performance.

The traditional approach to the integration of fractional-order starting value issues, according to [9], is based on the Caputo derivative, whose beginning conditions are employed to build the classical integral equation. The authors show, using a simple counter example, that this technique results in incorrect free-response transients. The frequency-distributed model of the fractional integrator and its distributed beginning conditions are used to solve this fundamental problem. They answer the preceding counter-example using this model and provide a methodology that is a generalization of the integer-order approach. Finally, in the linear situation, this technique is used to model Fractional Differential Systems (FDS) and for the formulation of their transients. Two expressions are constructed, one based on the Mittag–Leffler function and the other on the notion of a distributed exponential function.

According to [10], fractional-order differential equations are effective tools for modeling dynamic systems with long-term memory effects. The verified simulation of such system models using interval tools enables the computation of assured enclosures of attainable pseudo-state regions over a finite time horizon. In prior work, the author published an iteration method based on Picard iteration that uses Mittag–Leffler functions to determine guaranteed pseudo-state enclosures. In this study, the corresponding iteration is generalized to use exponential functions during the iteration scheme evaluation. A validated solution of integer-order sets of differential equations yields such exponential functions. The goal of this work is to show that using exponential functions for Mittag–Leffler functions instead of pure box-type interval enclosures not only improves the tightness of the calculated pseudo-state enclosures, but also minimizes the required computational cost. These claims are supported by a realistic simulation model of the charging/discharging kinetics of Lithium-ion batteries.

Finally, Ref. [11] investigates the synchronization of fractional-order uncertain delayed neural networks with an event-triggered communication strategy. By developing an appropriate Lyapunov–Krasovskii functional (LKF) and inequality approaches, sufficient criteria for the stability of delayed neural networks are obtained. The criteria are expressed as linear matrix inequalities (LMIs). To accomplish synchronization, a controller is derived using the drive-response idea, the LMI technique, and the Lyapunov stability theorem. Finally, numerical examples are provided to validate the effectiveness of the major findings.

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