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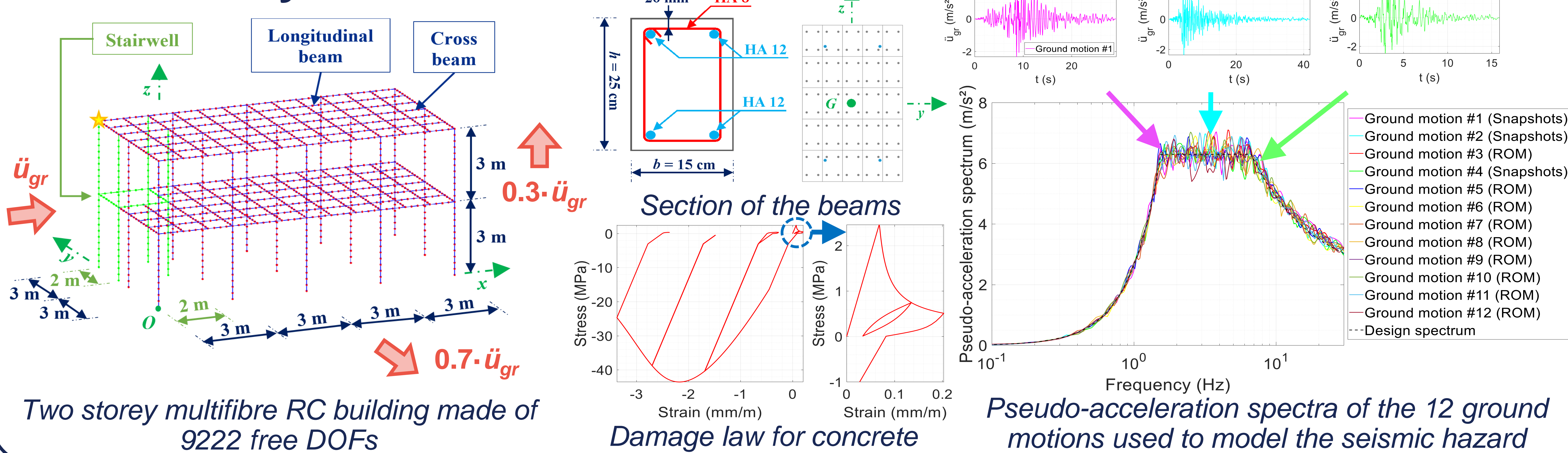
# POD-UDEIM MODEL ORDER REDUCTION IN THE CASE OF DAMAGING RC STRUCTURES SUBJECTED TO EARTHQUAKES

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**Earthquake Engineering, Reinforced Concrete, Multifibre Beam Elements, Reduced Order Model (ROM), Proper Orthogonal Decomposition (POD), Unassembled Discrete Empirical Interpolation Method (UDEIM)**

Earthquake engineering hybrid testing requires the use of efficient nonlinear ROMs that can be run in real time. To tackle this problem, a POD-UDEIM hyper reduced order model is here associated to a non-iterative  $\alpha$ -OS time-integration scheme to speed up FEM analyses on a two-storey RC structure made of damaging multifibre beam elements. In this work, the seismic hazard is modelled by a set of 12 earthquakes fitting the same design spectrum.

## Case study



Full order FEM analyses with ground motions #1, #2 and #4

## Snapshots

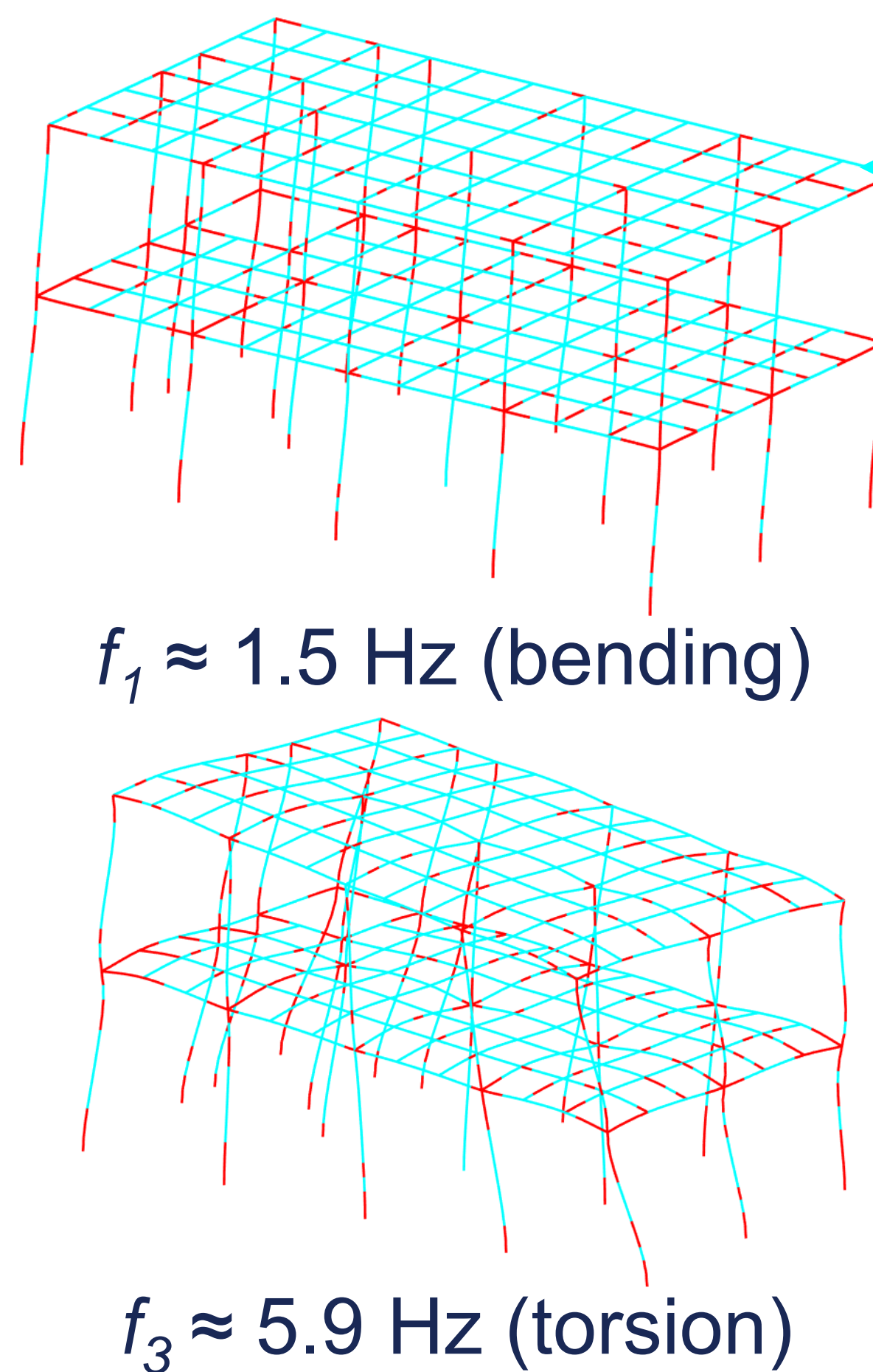
## Non iterative $\alpha$ -OS time scheme

$$M \cdot \ddot{u} + C \cdot \dot{u} + K_E \cdot u + \tilde{r}_{NL}(\tilde{u}) = -M \cdot \Gamma \cdot \ddot{u}_{gr}(t)$$

Explicit prediction of the nonlinear part of the restoring force vector

SVD on a set of  $N$  snapshots of  $u$

## POD-modes



$$u \approx B \cdot q$$

Projection of the displacements on a basis  $B$  made of  $n$  POD-modes

SVD on a set of  $N$  snapshots of  $\tilde{r}_{NL}$

## UDEIM-modes

Domain where the restoring forces are interpolated - 1295 elements

Reduced integration domain (RID) - 417 elements

Basis made of  $m$  UDEIM-modes

$$A = \Psi \cdot (P^T \cdot \Psi) +$$

Partition matrix related to all the components of the RID

Interpolation operator

$$\tilde{r}_{NL,u} \approx A \cdot \tilde{r}_{NL,u}^{RID}$$

Interpolation of the nonlinear part of the unassembled restoring force vector on the full domain

## Online phases

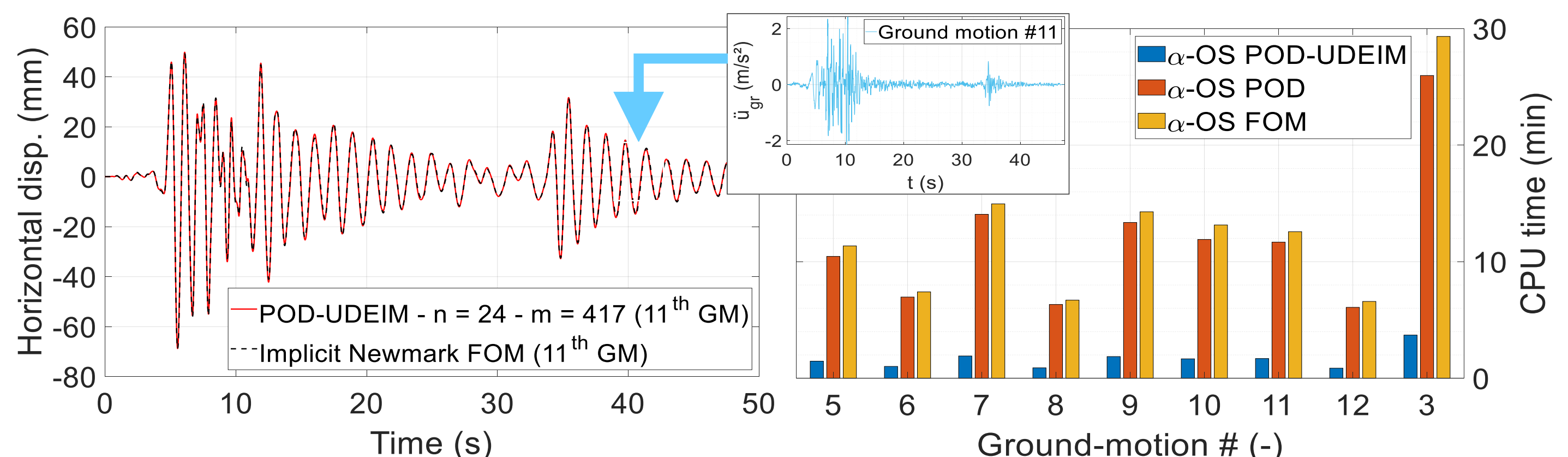
$$1 - \frac{\sum_{i=1}^n \lambda_i}{\sum_{i=1}^N \lambda_i} \leq \varepsilon$$

Singular value related to the  $i$ <sup>th</sup> mode

POD :  $\varepsilon = 1 \cdot 10^{-2} \rightarrow n = 24$

UDEIM :  $\varepsilon = 1 \cdot 10^{-2} \rightarrow m = 417$

Criterion used to set the number of POD and UDEIM modes



Time savings and error with POD and POD-UDEIM ROM

Ground motion	#3	#5	#6	#7	#8	#9	#10	#11	#12
Error/Implicit NM (%)	0.2	0.1	0.2	0.3	0.2	0.3	0.3	0.2	0.6
Time saving POD (%)	11	8	6	6	6	6	10	7	8
Time saving POD-UDEIM (%)	87	87	86	87	87	87	87	87	87

The error is less than 0.6 % with a 87 % time saving during each online phase.

Reduced order FEM analyses with ground motions #3 and #5 to #12

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